OR 842: ALTERNATIVE SYSTEMS OF PROBABILISTIC REASONING
FALL 2012 SYLLABUS
INSTRUCTOR: DAVE SCHUM

PLEASE NOTE: Although this course carries an OR [Operations Research] number, it is open to all graduate students in the Volgenau School of Engineering. The reason is that most research efforts on the part of Masters and PhD students in the Volgenau School of Engineering will involve probabilistic issues. These issues may require interpretations of probability that are not commonly encountered in conventional courses in probability. In this course we will consider an array of different views of probability that are not only interesting, but are also necessary on many occasions.

THE BASIC SUBJECT MATTER OF THIS COURSE

Conclusions drawn from evidence in any context are <u>necessarily</u> probabilistic; here are five reasons why this is so. Our evidence is always <u>incomplete</u>; we can never have all of it. Evidence is commonly <u>inconclusive</u> in the sense that it could mean more than one thing or is consistent with more than one possible conclusion. Evidence is frequently <u>ambiguous</u>, meaning that we cannot be certain what it is telling us or exactly what information it is conveying. When we have masses of evidence to consider, we will encounter patterns of <u>dissonance</u> in which some evidence points in one direction and other evidence points in different directions. Finally, evidence is <u>not perfectly credible</u> since it comes from sources having any gradation of credibility or believability shy of perfection.

Now, here comes a problem when we consider all of these evidential characteristics. There is no single view of probability that captures all five of these characteristics. In this course we will basically consider four alternative views of probabilistic reasoning. Each of these views adds insight into the complexities of probabilistic reasoning, but no single view says all there is to be said. Among the views we will discuss are the **Conventional or Bayesian view**, **Belief Functions,** the **Baconian** view, and **Fuzzy** systems for probabilistic reasoning. Each one of these formal systems, as well as others to be mentioned, has interesting and valuable properties that deserve our attention. The necessity for considering alternative views seems evident since it seems far too much to expect that any one formal system of probability can capture all of the richness of probabilistic reasoning. We will examine grounds for the necessity of considering alternative views of probabilistic reasoning and, in the process, consider applications of each of these views in non-trivial inference problems. Also of interest to us are issues concerning what seems to constitute "rational" or “optimal” probabilistic reasoning.

Since evidence forms the foundations of all probabilistic reasoning, it is vitally necessary to consider the various recurrent forms and combinations of evidence as well as the different uses of evidence in probabilistic inferences. <u>All</u> evidence has three major credentials or properties: <u>relevance</u>, <u>credibility</u>, and <u>inferential force or weight</u>. Relevance addresses the question: so what?, or how is this evidence linked to what we are trying to prove or disprove? Credibility addresses the question: can we believe what this evidence is telling us? The inferential force or weight of evidence addresses the question: how strong is this evidence in favoring or disfavoring possible conclusions being entertained?

It’s true of course that no evidence comes to us with these three credentials already supplied. They have to be established by defensible and persuasive arguments. This process is often difficult even for single items of evidence. The reason is that arguments from evidence to possible conclusions or hypotheses consist of often-long chains of reasoning in which each link in the chain corresponds to a source of doubt or uncertainty imagined by the person constructing the argument. But this argument process gets frighteningly complex when we have masses of evidence to consider. In such cases we have what are termed complex <u>inference networks</u> consisting of many, possibly interrelated, chains of reasoning. We will dwell on methods for constructing arguments from masses of evidence encountered in inference networks; such arguments require imaginative as well as critical reasoning. As we will see, careful argument construction can reveal the existence of an array of evidential and inferential complexities that can be exploited, if they are identified. A very wise person, whose works we will consider, has correctly said that probability is more about arguments than it is about numbers.

As you see, probabilistic inference involves three major ingredients: evidence, hypotheses or possible conclusions, and arguments. It's true that in classroom exercises some or all of these ingredients are usually supplied for you. But in research you plan to do yourself, say for your Masters thesis or PhD dissertation, it will be up to you to discover or generate these ingredients yourself. This discovery process requires imaginative reasoning on your part. This would not be an adequate course in probabilistic reasoning if it did not dwell on the imaginative elements of such reasoning. All discovery processes in any context require the asking of questions. It might be thought that the formal or mathematical probabilistic systems we will consider can play no role in discovery. This is not true however. So often a well-considered probabilistic analysis can alert you to ask questions you might not have thought of asking if you had not done this analysis. I will give you several examples of how probabilistic analyses allowed me to generate new lines of inquiry I would not have imagined if I had not done this analysis.

TEXTUAL AND OTHER REFERENCES ON PROBABILISTIC REASONING:

1). There are many relevant and interesting works on the topic of probabilistic reasoning to be found in a variety of disciplines; our lives are simply too short to master all of them. One trouble is that most published works on the topic of probabilistic reasoning do not provide much discussion of the very basis for it, namely **evidence**. Many of the works we will consider focus just upon algorithms for combining probabilistic assessment of various sorts and say very little about the evidential foundations for such assessments. After offering this seminar for a number of years, and at the suggestion of many students who completed it, I decided to write a book that covers what I regard as the basic evidential foundations of each of the formal systems of probability we will discuss. This book is entitled: *The Evidential Foundations of Probabilistic Reasoning* [Northwestern University Press, Evanston, IL. 2001; paperback edition]. Some of the chapters of this book are based upon notes I gave students in previous offerings of this course. In addition, four former students provided editorial and other comments on this work to ensure that its contents would be especially relevant to the interests of graduate students in our school of engineering at GMU. In asking you to purchase the book just mentioned, I encounter an aversion I have always had. I am greatly averse to pocketing the royalties I would obtain from students who I ask to buy this book. Consequently, I will refund to you the royalty I receive from this book. The only royalty I hope to obtain is your telling me that this book was readable, interesting, and that it told you various things about probabilistic reasoning you did not already know.

2). There are, of course, many other references to consider; these references come from many different disciplines and frequently involve matters about which there is not widespread awareness. At the end of this syllabus is an expanded reference list containing some basic references that are keyed to the major probability systems we will discuss in this seminar.

3). During this seminar I will provide you with an assortment of notes and handouts on particular topics. In most cases these notes will consist of applications of the various probabilistic reasoning systems and additional sensitivity analyses of the sort you will find in the assigned text noted above.

TOPICS AND READING ASSIGNMENTS

Following is the sequence of topics we will cover in this seminar together with associated reading assignments from: *Evidential Foundations of Probabilistic Reasoning* [EFPR]. The dates shown are only approximate; we may wish to dwell longer on some topics than on others.

I. <u>SOME PRELIMINARY MATTERS [29 August]</u>

I will begin by describing two basic conditions in which concern about probabilities arises. The first I will describe is the Enumerative Condition in which we determine probabilities, or estimates of them, by counting the past occurrence of events of interest. There are two cases to consider here: games of chance, and statistics. It turns out that the conventional view of probability we all learn about in school assumes the enumerative condition. Such conditions are repeatable or replicable in which we can observe the same processes over and over again to see how many times some event of interest has occurred. We all learn about three axioms that apply in enumerative conditions: (i) probabilities are positive numbers or zero; (ii) a sure or certain event has probability 1.0; and (iii) the probability of mutually exclusive events taken together is the sum of their separate probabilities. We also learn that these three axioms were first proposed by the Russian mathematician A. N. Kolmogorov in 1933. These three axioms also apply to conditional probabilities and, as a result, Bayes' rule is consistent with them.

But if the Enumerative view of probability was the only situation to which probability could apply, we would be out of luck in applying probability to so many other situations in which we have natural concerns about uncertainties. In these situations we encounter singular, unique, or one-of-a-kind events and have concerns about their likeliness. If these events have occurred in the past, they have only done so, or not, on exactly one occasion. If they occur in future, they will only occur, or not, on exactly one occasion. I will refer to such situations as the Non-Enumerative Condition because there is never anything to count. Such conditions regularly occur in law, intelligence analysis, medicine, history, business, and so many other situations.

Now, a very good question is: do the rules and properties of enumerative probability also apply in non-enumerative cases? In some of these non-enumerative cases probabilities can only be assessed by subjective human judgments. On one view we will discuss, the Subjective Bayesian View, the answer to the above question is *yes*, provided that these judgments obey enumerative rules. But the three other views of probability will say that the answer to the above question is *no*; different rules are required in non-enumerative cases. In the case of Belief Functions, the enumerative additivity axiom [axiom (iii) above] is violated and we can have beliefs for mutually-exclusive events that are not additive. There are many other unique characteristics of belief functions we will discuss. The Baconian system adopts an eliminative and variative view of probabilistic inference that goes back to the views of Sir Francis Bacon. On this view, a matter of great concern is: how complete is the evidential coverage of relevant variables involved in the probabilistic inference at hand? The Fuzzy view of probability acknowledges the vagueness or imprecision that so often exists in the ingredients of inference as well as in our judgments. In so many situations, probabilistic judgments are given in words rather than in numbers. After I give you this brief account of alternative systems we will study in detail, I will show you how successful these systems are in capturing the five characteristics of evidence I mentioned in the very first paragraph of this syllabus.

To further set the stage for later discussions, I will give you a bit of history of the development of interest in the evidential foundations of probabilistic reasoning. In the process I will try to convince you that any theory of probabilistic reasoning must be concerned about two major issues: (i) structural matters in the generation of arguments from evidence to hypotheses/possibilities, and (ii) matters concerning the assessment and combination of ingredients associated with the force, strength, or weight of evidence on these hypotheses/possibilities. Further, I will elaborate a bit on my claim that human reasoning in natural settings [the "real world", if you like] involves mixtures of three forms of reasoning I will mention: deduction, induction, and abuction. I will also give you an overview of the literature that exists in so many disciplines that is relevant to an understanding of the complexity of probabilistic reasoning.

II. EVIDENCE, PROBABILITY, AND STRUCTURAL ISSUES [5 - 12 September].

When most people think about probability they think only about numbers. But, as I mentioned, probability theories are equally concerned about the arguments that are constructed to support the use of numbers. The construction of arguments is a creative task concerning which most of us have very little appropriate tutoring. This is especially true when we must construct arguments from masses of evidence. We will consider some exercises showing just how difficult this can be. One major benefit of studying structural matters is that it allows us to discern various recurrent forms and combinations of evidence and to observe how evidence is used during the process of probabilistic reasoning, regardless of the view of probability one adopts. To be useful in probabilistic reasoning, evidence must have certain credentials concerning its relevance, credibility, and inferential force, strength, or weight. As I mentioned, no evidence comes to us with these credentials already established; they must be established by cogent arguments. The major issue that divides the alternative views of probabilistic reasoning we will examine involves how best to grade the inferential force, strength, or weight of evidence and then to combine such gradings across many items of evidence. Reading Assignment: EFPR Chapters 1 - 3.

III. STRUCTURAL ISSUES AND COMPLEX INFERENCE [19 - 26 September]

There is presently a substantial level of research on what are called inference networks. As I will mention, inference networks can have different forms and will suit different purposes. In a wide variety of applied areas, people have the difficult task of trying to make sense out of masses of evidence that, upon examination, reveal all of the forms and combinations of evidence we will have considered in Section II. When arguments are constructed from a mass of evidence they begin to resemble complex networks. Many [not all] of the important elements of complex inference can be captured in graph theoretical terms. Naturally, there are many current attempts to develop various forms of computer assistance in coping with the complexity of inference based on masses of evidence. Matters we discuss here will be quite relevant to other courses now offered in our school of engineering concerning inference networks [e.g. Professor Kathy Laskey's OR/STAT 719].

Reading Assignment: EFPR Chapter 4

IV. ALTERNATIVE PROBABILITY SYSTEMS I: BASIC THEORETICAL ELEMENTS [3 – 31 October]

Here we come to a discussion of the different views about probabilistic reasoning. If only for historical reasons, we begin with the conventional or Bayesian system for probabilistic reasoning. Then we will consider the Shafer-Dempster theory of belief functions, Cohen's system of Baconian probability for eliminative and variative probability, and Zadeh's ideas on fuzzy reasoning and fuzzy probabilities. Our basic objective here is to examine what one is committed to when one applies any of these formal systems. A question frequently asked is: which one of these systems do you prefer? This is rather like asking whether you prefer your saw or your hammer. Each system "resonates" to particular important elements of probabilistic reasoning. Each system has something important to tell us about probabilistic reasoning, but no system says all there is to be said. At this point we will dwell upon what is meant by the term "rational" probabilistic reasoning. In this section we will also give attention to the matter of combining probabilistic and value-related judgments in situations in which probabilistic reasoning is embedded in the further process of choice.

Reading Assignment: EFPR Chapters 5 - 6

V. ALTERNATIVE PROBABILITY SYSTEMS II: EXAMPLES AND APPLICATIONS [7 - 14 November. No class on 21 November]

Having examined the theoretical underpinnings of alternative systems of probabilistic reasoning, we will examine what each produces in applications in different inferential contexts in which different forms and combinations of evidence are encountered. In some cases, our examination will consist of sensitivity analyses performed on probabilistic expressions designed to capture various forms and combinations of evidence. It is in this process that we begin to observe how many important and interesting evidential and inferential subtleties lie just below the surface of even the "simplest" of probabilistic reasoning tasks. These subtleties, if recognized, can be exploited in our probabilistic reasoning.

Reading Assignment: EFPR Chapters 7 - 8

VI. DISCOVERING THE INGREDIENTS OF PROBABILISTIC REASONING [28 November- 5 December]

As I noted, the ingredients of probabilistic reasoning are rarely provided for us [except in classroom exercises and examples]; they have to be discovered or generated. Stated in other words, there is the necessity for imaginative or creative reasoning in every probabilistic reasoning task. What we have time to do is simply to examine some of the imaginative elements of probabilistic reasoning. The topics of discovery and imaginative reasoning are of sufficient importance that we devote an entire seminar to the topic [SYST 944] that is offered in the Spring semester.

Reading Assignment: EFPR Chapters 9-10

PROCEDURAL MATTERS AND METHOD OF EVALUATION

To tell you how I believe this seminar should proceed, I will make reference to the thoughts of Sir Francis Bacon. Bacon argued that, in any scholarly activity, reading makes us **full**, discourse makes us **ready**, and writing makes us **accurate**. We will all do a fair amount of each in this seminar. As far as **reading** is concerned, you have one major text and the various handouts and other materials I will give you. In addition, the reference list I include at the end of this syllabus contains a wealth of information on all of the probability systems we will discuss. You can pick and choose among these references depending upon where your interests take you.

As far as **discourse** is concerned, I look upon this seminar as an experience in the sharing of ideas. Indeed, my view of scholarship is that it is a sophisticated form of sharing; we learn from each other. I have no wish to monopolize our discussions. On each occasion we meet I will try to get things started and will bring various matters to your attention. What happens after this depends upon you. Often, you may believe I am saying outrageous things with which you disagree; if so, your task is then to show me how I have been misled. In addition, you may have questions that I have not raised; your task is to raise them as we proceed. In short, I hope our meetings will be both enjoyable and productive; whether or not this happens depends upon all of us.

Finally, the topic of **writing** brings us to the first of two methods of evaluating your progress in this seminar. We will encounter literature from a number of different disciplines; no one discipline dominates scholarship on probabilistic reasoning. Together, the various works we will discuss supply a breadth of view. Depth of view is provided by your focus upon one or more of the formal systems; such focus will form the basis for a paper you will write, which will count 75% of your grade. The choice of focus is yours, provided that it concerns **probabilistic** reasoning or some matter that directly involves such reasoning. Perhaps many of you are now engaged in outside work to which the essential ideas in this seminar are relevant. In the past I have been quite free in allowing students to report on their efforts to apply ideas from this seminar to some ongoing project of interest in their outside work. Sadly, only a small number of these papers have been of a level of quality one expects from doctoral students. One reason is that students frequently spend too much time dwelling upon substantive issues in the problem domain and too little time on the theoretical probabilistic matters that are being applied in this domain. In some cases there was very little evidence in a paper that the student actually mastered any probabilistic matters that could have potential application. I certainly hope and expect that there will be ideas in this seminar that have applicability in areas of interest to you. If you do attempt to apply ideas from this seminar to particular ongoing work you are doing, I will be most concerned about how well and how completely you have mastered the ideas you are attempting to apply. Finally, you should look upon this written work as something that you do for yourself, not for me. It will simply demonstrate that you have attempted to acquire some **uncommon** understanding of a probabilistic inferential issue that captures your interest.

The remaining 25% of your grade will be based upon some exercises and problems that I will assign as we proceed. I expect to give you three such exercises during the semester. Many of these exercises will involve probabilistic analyses of various forms and combinations of evidence that appear in your text and in our class discussions.

AN EXPANDED REFERENCE LIST

Following is a list of basic references to works in each of the alternative systems of probabilistic reasoning we will discuss. In addition, I have included one or more important references for each system that are frequently cited but, with less frequency, actually read. I will have more references for you as we proceed.

I. Views Based Upon The Conventional Calculus Of Probability.

Perhaps the most frequently cited but infrequently read item on our list is the original paper by Thomas Bayes that was communicated by Richard Price to the Royal Society Of London two years after Bayes' death.

1) **An Essay Towards Solving A Problem In The Doctrine Of Chances.** By The Late Rev. Mr Bayes, F. R. S. Communicated by Mr. Price in a letter to John Canton, A. M., F. R. S., **Philosophical Transactions Of The Royal Society**, pp 370-418, 1763.

There is now debate about whether or not what has become known as Bayes' Rule should, in fact, be attributed to Bayes. Here is a paper on the idea that others may have earlier tumbled to the essence of Bayes' Rule.
2) Stigler, S. M., Who Discovered Bayes's Theorem ? **The American Statistician**, November 1983, Vol. 37, No. 4, p 290 - 296.

Here are some tutorials on the use of Bayes' rule in probabilistic inference.
3) von Winterfeldt, D., Edwards, W., **Decision Analysis And Behavioral Research**, Cambridge, Cambridge University Press, 1986, Chapters 5 and 6.
4) Schum, D., **Evidence And Inference For The Intelligence Analyst** [Two Volumes], Lanham, Md., University Press Of America, 1987. Chapters 6, 7, and their supplements.

There are now many references relative to the use of Bayes' Rule in **statistical** inference. Here are two places to start if you have interest in statistical inference.
5) Winkler, R., **Introduction To Bayesian Inference And Decision**, New York, Holt, Rinehart, & Winston, 1972.
6) O'Hagen, A., **Probability: Methods And Measurement**, London, Chapman & Hall, 1988.

Here are three works that, to varying degrees, consider Bayes' rule as the normative standard for probabilistic reasoning:
9) Good, I. J. **Good Thinking: The Foundations of Probability and Its Applications**. University of Minnesota Press, Minneapolis. MN, 1983
8) Earman, J., **Bayes or Bust ?: A Critical Examination of Bayesian Confirmation Theory**, Bradford Books, MIT Press, 1992
9) Howson, C., Urbach, P., **Scientific Reasoning: The Bayesian Approach**, Open Court Press, 1989.

Here are some works on the application of Bayes' rule in complex "inferential networks"
10) Pearl, J., **Probabilistic Reasoning In Intelligent Systems: Networks Of Plausible Inference**, San Mateo Calif., Morgan, Kaufman Publishers, 1988
11) Schum, D., **Evidence And Inference For The Intelligence Analyst** [Two Volumes], Lanham, Md., University Press Of America, 1987. Particularly Chapters 4, and 8 through 12 and their supplements.

12) Kadane, J., Schum, D. **A Probabilistic Analysis of the Sacco and Vanzetti Evidence**. New York, Wiley, 1996 [A Bayesian analyses of the evidence in a celebrated murder trial]
13) Glymour, C. **The Mind's Arrows: Bayes Nets and Graphical Causal Models in Psychology**. MIT Press, Cambridge, MA, 2001
14) If you wish to read more about Thomas Bayes himself and see his actual works on probability and on other matters, see: **Dale, A. Most Honorable Remembrance: The Life and Works of Thomas Bayes. Springer-Verlag, New York, 2003**.

We should note here that there is a very large literature on various possible interpretations of numbers that correspond to the standard or conventional calculus. Indeed, this is an entire topic by itself. There are two useful summaries of alternative conceptions of a conventional probability, they are:
15) Fine, T., **Theories Of Probability: An Examination Of Foundations**, New York, Academic Press, 1973
16) Weatherford, R., **Philosophical Foundations of Probability Theory**, London, Routledge & Kegan Paul, 1982

II. A Theory Of Nonadditive Beliefs Based On Evidence.

It is easily shown how probabilities encountered in **aleatory** [games of chance] and **frequentistic** [statistical] contexts can be trapped within the conventional probability calculus. But what about the array of **epistemic** contexts in which numbers are used to grade the **strength of our beliefs** about whether or not some event has happened, is happening, or will happen ? In such contexts we often encounter singular, unique, or nonreplicable events that can have no frequentistic interpretation. In epistemic contexts many commonly-encountered credal or belief states cannot easily be trapped within the bounds of the conventional system; it turns out that this has been known for centuries. Our first reference here is a very useful and well-done treatise on the history of the concept of probability. Discussed in this treatise are some of the difficulties the conventional calculus has experienced in the trapping of these credal states.
1) Hacking, I., **The Emergence Of Probability**, Cambridge, Cambridge University Press, 1978.

There are alternatives to the use of Bayes' rule in the task of combining our beliefs based on some emerging body of evidence; this has also been recognized for centuries. One mechanism for belief combination has been termed "Dempster's rule"; this rule has roots in much earlier work. In 1976 Glenn Shafer took Dempster's rule as the cornerstone for a "new" system of probabilistic reasoning involving what he terms a "belief function". We shall term this species of probabilistic reasoning the "Shafer-Dempster" system.

<u>A. The Shafer-Dempster View</u>.

Shafer was a student of Dempster's and has now achieved a considerable measure of fame as a result of the following work:
2) Shafer, G., **A Mathematical Theory Of Evidence**, Princeton, N.J., Princeton University Press, 1976.
Some of us believe this work is not actually a theory of evidence but a theory of belief based upon evidence; form your own opinion as we discuss his work and the work of others. Here is a summary of recent thinking about the Shafer-Dempster system of belief functions:

3) Yager, R., Fedrizzi, M., Kacprzyk, J. **Advances in the Dempster-Shafer Theory of Evidence.** Ney York, John Wiley & Sons, 1994.
Here are several often cited works by Glenn Shafer that elaborate on the historical foundations of the Shafer-Dempster view and that also are critical of any view in which Bayes' rule is advocated as **the** "normative" or "prescriptive" view of probabilistic reasoning.
4) Shafer, G., Bayes's Two Arguments For The Rule Of Conditioning, **The Annals Of Statistics**, Vol. 10, No. 4, 1982.
5) Shafer, G., Conditional Probability, **International Statistical Review**, Vol. 53, 1985
6)Shafer, G., The Combination Of Evidence, **International Journal Of Intelligent Systems**, Vol. 1, 1986.

7) Shafer, G., The Construction of Probability Argument, **Boston University Law Review**, Vol. 66, Nos, 3 & 4, May/July, 1986.
8) Shafer, G., Pearl, J., (eds), **Readings in Uncertain Reasoning**, Morgan Kaufmann Publishers, 1989 [compares Bayes with Shafer-Dempster. Formerly assigned as a text in this seminar].

B. "Potential Surprise": Another Nonadditive System.

More than one person has been interested in relationships between the concepts of **probability** and **possibility**; one such person is the British economist G. L. S. Shackle. In the following work Shackle proposed a metric, called **potential surprise**, for grading our beliefs about the possibility of events, something he believed was not possible within the conventional calculus.
9) Shackle, G. L. S., **Decision, Order, And Time In Human Affairs**, Cambridge, 1969.
This system is still being discussed, thanks to the efforts of the American philosopher Isaac Levi. Levi claims this system to be quite general and sees in it some parallels with the Shafer-Dempster system. In some quarters this system of reasoning is referred to as the Levi-Shackle system of reasoning. Here are some further, more up to date references to potential surprise.
10) Levi., I., **Decision And Revision: Philosophical Essays On Knowledge And Value**, Cambridge, Cambridge University Press, 1984 [Chapter 14].
11) Levi, I., **The Enterprise Of Knowledge: An Essay On Knowledge, Credal Probability, And Chance.** Boston, MIT Press, 1983

III. Probability In Eliminative and Variative Induction.

In many contexts, science for example, we subject our hypotheses to a testing process in which only the fittest survive. In such tests evidence is used as a basis for **eliminating** hypotheses. As Professor L. Jonathan Cohen [Queens College, Oxford] will tell us in his works, a particular hypothesis seems to have increasing probability or provability as it survives our best efforts to invalidate it. We subject hypotheses to a variety of different evidential tests; the more of these tests some hypothesis survives, the more confidence we have in it. The key word here is **variety**; the survival of any hypothesis depends upon the extent to which it holds up under **different** conditions. Replication of test results is important but we cannot gather support for some hypothesis simply by performing the same test over and over again. Drawing upon the work of Sir Francis Bacon and John Stuart Mill, Professor Cohen has given us a system of probability that is suited to what he terms **eliminative and variative** inductive inference. In this system of "Baconian" probabilistic reasoning, probabilities grade the extent to which some hypothesis survives an eliminative testing process, The "weight" of evidence, in Baconian terms, is related to the **number** of evidential tests we perform and to the extent to which our tests cover variables relevant in discriminating among the hypotheses we consider. Cohen's system is the only system that specifically grades the **completeness or the sufficiency** of our evidence. How likely we view some hypothesis depends upon how many relevant questions concerning our hypotheses that our existing evidence does not answer.

The first reference is to Cohen's work on developing means for grading the **inductive support** that evidence provides in the eliminative testing process Cohen describes.
1) Cohen, L. J., **The Implications Of Induction**, London, Methuen & Co. Ltd., 1970.
Cohen certainly acknowledges that there is room for more than one view of probabilistic reasoning. Cohen's "polycriterial" account of probability is given in the following three references, the second of which can probably be termed his major work.
2) Cohen. L. J., Probability: The One And The Many, **Proceedings Of The British Academy**, V ol. LXI, 1975.
3) Cohen. L. J., **The Probable And The Provable**, Oxford, The Clarendon Press, 1977. This is Cohen’s major work on Baconian probability.
4) Cohen, L. J., **An Introduction to the Philosophy of Induction and Probability**, Oxford University Press, 1989. I used to require every student to read this work; it is truly outstanding. Unfortunately, it is now out of print.

Here are several other of Cohen's works that are of particular importance to anyone seeking to understand the full dimensions of Cohen's views.

5) Cohen, L. J., Bayesianism Versus Baconianism In The Evaluation Of Medical Diagnosis. **British Journal For The Philosophy Of Science**, Vol. 31, 1980.
6) Cohen, L. J., Twelve Questions About Keynes's Conception Of Weight, **British Journal For The Philosophy Of Science,** Vol. 37, 1985.
7) Cohen, L. J., Hesse, M., **The Applications Of Inductive Logic**, Oxford, The Clarendon Press, 1980.
8) Cohen, L. J., **The Dialogue Of Reason**, Oxford, The Clarendon Press, 1986.

Here are three references in which Cohen's Baconian system is discussed and compared with other views.

9) Schum, D., A Review Of A Case Against Blaise Pascal And His Heirs, **University Of Michigan Law Review**, Vol. 77, No. 3, 1979.
10) Schum, D., Probability And The Processes Of Discovery, Proof, And Choice. **Boston University Law Review,** Vol. 66., Nos. 3 & 4, May/July 1986.
11) Schum, D., Jonathan Cohen And Thomas Bayes On The Analysis Of Chains Of Reasoning. In: **Rationality And Reasoning: Essays In Honor Of L. Jonathan Cohen** [eds. Eells, E., Maruszewski, T.] Amsterdam, Rodopi, 1991.

IV. Imprecision, Fuzzy Probabilities, And Possibilities.

The Pascalian system of probability is rooted in a system of two-valued logic; a statement is either true or false or a particular element is either in some subset or it isn't. In 1965 Professor Lotfi Zadeh argued that our inferences and decisions are often based upon information that is **imprecise or ambiguous** and for which this two-valued logic is inappropriate. He coined the term "fuzzy sets" to describe collections of elements with indistinct, imprecise, or "elastic" boundaries. Zadeh has argued that a different calculus is necessary to represent reasoning based upon fuzzy information. His early work has generated enormous enthusiasm and there are now over 20,000 papers, books, and other materials that have been published on fuzzy matters since 1965, the year in which Zadeh's first work on fuzzy sets saw the light of day.

1) Zadeh, L., "Fuzzy Sets", **Information And Control**, Vol. 8, 1965.

Zadeh and his now enormous international body of followers have published papers on the application of fuzzy sets in an array of inferential and decisional contexts. The best single collection of Zadeh's papers is found in the following.

2) Yager, R., Ovchinnikov, S., Tong, R., Nguyen, H., **Fuzzy Sets And Applications: Selected Papers By L. A. Zadeh**, New York, Wiley & Sons, 1987.

Here are several fairly current works that may be regarded as tutorial regarding fuzzy sets and systems.

3) Klir, G., Folger, T., **Fuzzy Sets, Uncertainty, and Information**, Prentice Hall, 1988
4) McNeill, D., Freiberger, P., **Fuzzy Logic**, Simon & Schuster, 1993
5)Kosko, B., **Fuzzy Thinking: The New Science of Fuzzy Logic**, Hyperion, 1993. This is an absorbing (but frequently irritating) work by the leading spear-carrier of fuzzy reasoning.

The following work is critical of the idea that fuzzy logic actually extends classical logic.

6) Haack, S. **Deviant Logic, Fuzzy Logic: Beyond the Formalism**. University of Chicago Press, 1996.

V. Probabilistic Inference And Its Role In Decisions

As noted in your syllabus, we should give attention to inferential activity in the following two contexts. In many situations, various areas of science for example, inferential activity is simply part of the process of knowledge-acquisition. However, in other situations such as in law, medicine, intelligence analysis, and so on, inferential activity is embedded in the further process of choice. In all of these other situations, assessments of probability have somehow to be combined with assessments of the value of consequences that occur to us when we contemplate various choices in the face of uncertainty. There is a substantial literature on the combination of inferential and value-related ingredients in choice under uncertainty. Two quite different views of this process of combination are found in the following three references.

1) von Winterfeldt, D., Edwards, W., **Decision Analysis And Behavioral Research**, Cambridge, Cambridge University Press, 1986. [See Chapters 1, 2, 3]
2) Lindley, D., **Making Decisions**, [2nd ed], London, Wiley & Sons, 1985.
3) Shafer. G., Savage Revisited, **Statistical Science**, Vol. 1, No. 4., 1986.

LAST BUT NOT LEAST, WHERE TO FIND YOUR INSTRUCTOR

I usually lurk in the vicinity of Room 2226, Engineering Bldg. II; my office phone is 703-993-1694. If I am not in my office, I am almost certainly at home: 2219 Chestertown Dr., Vienna, Va. My home phone is: 703-698-9515. Please do not hesitate to contact me at either of these locations. My preferred e-mail address is dschum398@earthlink.net , You can also reach me at <dschum@gmu.edu>. I will do all I can to make this seminar useful and enjoyable for you.