# What is Operations Research?

- Morse and Kimball, 1946:
  - “Operations research has come to describe the scientific, quantitative study of the operations of war”
  - “Operations research … is the application of the scientific technique to the study of combinations of men and equipment in warfare”
  - “Operations research is a scientific method of providing executive departments with a quantitative basis for decisions regarding operations under their control”
- The latter is the accepted definition today
- Common threads
  - Use of the scientific method
  - People in combination with equipment
  - Decision to be made

# A WW II Example - Submarine Contact Data

- Original data:

| | Distance from Shore | | | |
|---|---|---|---|---|
| | 0-60 | 60-120 | 120-180 | 180-240 |
| **Contacts** | 21 | 11 | 5 | 2 |

- **Result: panic**

- The story, after some analysis:

| | Distance from Shore | | | |
|---|---|---|---|---|
| | 0-60 | 60-120 | 120-180 | 180-240 |
| **Contacts** | 21 | 11 | 5 | 2 |
| **Flying Hours** | 15000 | 3700 | 600 | 170 |
| **Contacts/Hour** | 0.001 | 0.003 | 0.008 | 0.012 |

# The Scientific Method

- The classic scientific method (from Dr. Jose Wudka, a physicist from UC Riverside)
  1. Observe some aspect of the universe
  2. Invent a tentative description, called a hypothesis, that is consistent with what you have observed
  3. Use the hypothesis to make predictions
  4. Test those predictions by experiments or further observations and modify the hypothesis in the light of your results
  5. Repeat steps 3 and 4 until there are no discrepancies between theory and experiment and/or observation
- Intent is to eliminate bias, be repeatable and reproducible

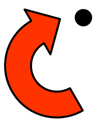
# OR Methodology (according to Winston)

- Winston (sec 1.1) translates this into a methodology:
  - Step 1: formulate (he should say *define*) the problem
  - Step 2: observe the system
  - Step 3: formulate a mathematical model of the problem
  - Step 4: verify the model and use it for prediction
  - Step 5: select a suitable alternative
  - Step 6: present the results and conclusions of the study to the organization
  - Step 7: implement and evaluate recommendations
- **This is idealistic (and unrealistic) when people are involved**

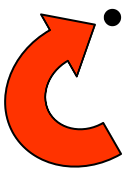
# My Typical OR Project Experience

- Receive tasking from intermediary

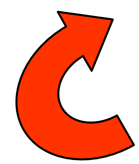
- Fight for audience with decision maker
  - Receive loose description of symptoms
  - Struggle over approval of problem definition (several iterations)



- Search for true expertise and useful information
  - Ferret out false experts and useless or misleading data
- Build initial model, which always fails
  - If not fired, build and test new model
- Garner support for emerging results
- Fight for acceptance from decision makers



- Work through implementation
  - Do *many* model rebuilds



**= at least 1 iteration**

# Don’t Be Fooled - OR is Powerful

- 2005 Edelman prize winners for OR practice:
  - Supplier Negotiation Process at Motorola
  - Routing Optimization for Waste Management
  - Supply Chain Improvements for Phillips Electronics
  - Aircraft Ownership Operations for Bombadier Flexjet
  - Decision Support System for Hong Kong International Terminals
  - Asset and Order Management for John Deere
  - US/Russia Plutonium Disposition Option Analysis
- All big problems, huge organizations, lots at stake
- OR methods allowed a *scientific basis* for decisions

# Resources for OR

- INFORMS - Institute for Management Sciences (www.informs.org)
  - Lead professional society for OR in the U.S.
  - Publishes many journals; offers spectacular student rates
  - Washington, D.C. chapter (http://winforms.chapter.informs.org)
- MORS - Military Operations Society
  - Requires security clearance
  - INFORMS Military Application Section does not, however
- Journals
  - Recommend *Interfaces, OR/MS Today* for those starting out
  - Recommend *Military Operations Research* for those doing defense work
  - Many other journals, with differing degrees of difficulty

# Learning About the People Part of OR

- Recommended texts:
  - The Mythical Man-Month (Brooks)
  - Up the Organization (Townsend)
  - The Masks of War (Builder - for those associated with DoD)
- INTERFACES online is a great resource
  - **http://interfaces.pubs.informs.org/**
  - Articles older than 3 years can be downloaded free
  - Recommend anything by R. E. D. (Gene Woolsey), Robert Machol, Andrew Vasonyi
- Also, see Doug Samuelson's ORacle column in OR/MS Today

# My Background

- 1980-2003: Active duty, USAF
- Presently: Senior Analyst, MITRE Corporation
- Degrees
  - BS, USAF Academy
  - MS, Rensselaer Polytechnic Institute
  - Ph.D., Naval Postgraduate School
- Career OR analyst (logistics, airlift, campaign modeling, special ops, weapons requirements, DoD strategy)
- Former board member, Military Operations Research Society
- Publications in various journals; referee for *Interfaces*, *MOR*, *Naval Research Logistics*
- Considerable experience in optimization, with several implemented large-scale models

# Course Admin

- Home page: http://www.seor.gmu.edu/syllabi/04F/OR541/OR541.htm
- Office hours: best time is before/after class
- Prefer to handle questions via email
  - “Only when one writes do the gaps appear and the inconsistencies protrude” - Frederick Brooks
  - Use OR541GMU@aol.com to send email to me
- Other details on course syllabus
  - Grading
  - Exam dates
  - Fundamental Rules
  - Philosophy

# Course Structure

- Concentrates on *deterministic optimization* models
  - Deterministic - no random variables
  - Optimization - finding the “best” solution among alternatives
- Some common deterministic models *not* covered
  - Dynamic programming
  - Game theory
  - Scheduling
  - Inventory control
- Will follow Winston, chapters 3-4, 6-10, 12

# Optimization, Programming

- Optimization
  - In general, trying to choose the best solution among competing alternatives
  - Addresses a common dilemma; competing interests chasing limited resources
  - Requires definition of:
    - “fitness” of a particular solution
    - knowledge of how the alternatives consume resources
    - availability of resources
- Why is it called programming?
  - The original motivation: find a way to scientifically construct the Air Force *program* (multi-year budget) in the late 40’s
  - Has nothing to do with computer programming

# History (mostly Dantzig, Orchard-Hays)

- 1947-49: Dantzig formulates general linear programming (LP) theory
- 1951: Karush-Kuhn-Tucker conditions begin nonlinear programming
- 1952: commercial applications appear in oil industry
- 1954: first commercial-grade LP code written by William Orchard-Hays
- 1954-56: network flow theory began with Ford-Fulkerson
- 1955: first stochastic programming theory published
- 1958: Gomory develops first integer programming methods
- 1960-62: large-scale optimization begins with Dantzig-Wolfe, Gilmore-Gomory, and Benders' decomposition methods
- 1966: IBM introduces MPS/360, a complete LP package
- 1972: first theory on computational complexity
- 1978: Khachian publishes polynomial-time LP algorithm
- 1984: Karmarkar publishes *working* polynomial-time LP algorithm
- Late 1980's: commercial algebraic modeling languages arrive
- Today: can solve gigantic problems on a $3000 PC

# So, What is A Linear Program?

- Start with a typical linear algebra problem, **Ax = b**
  - **A** is a **m** (row) X **n** (column) matrix, **b** an **m**-vector (called the right-hand side), **x** is a vector of variables
  - As simple as:

$$a_{11}x_1 + a_{12}x_{12} = b_1$$

$$a_{21}x_1 + a_{22}x_{12} = b_2$$

- From linear algebra, if **m=n**, then this system has:
  - 1 solution, if? **(A is invertible)**
  - No solution if? **(A is singular and inconsistent)**
  - Infinite solutions if? **(A is singular and consistent)**

# What If n > m?

- We have more variables than equations
- Can form $\binom{n}{m}$ square matrices from A; many nonsingular
- How do we distinguish the alternatives?
- Answer: form an *objective function*
  - Notion unknown prior to Dantzig's work
  - Provided way to score solutions
- Dantzig proposed a linear objective
  - Define an **n**-vector, **c**
  - For any solution **x** of **Ax=b**, the "score" of the solution is **cx,** e.g.

$$c_1 x_1 + c_2 x_2$$

# What If the Problem Has Inequalities?

- If **b** represents available resources, we may not require that we use them all, e.g.

$$a_{11}x_1 + a_{12}x_{12} \le b_1$$

$$a_{21}x_1 + a_{22}x_{12} \le b_2$$

- Now what? The linear algebra book doesn't cover this!
- Solution: add *slack* variables to turn them into equalities

$$a_{11}x_1 + a_{12}x_{12} + s_1 = b_1$$

$$a_{21}x_1 + a_{22}x_{12} + s_2 = b_2$$

$$s_1, s_2 \ge 0$$

**Note restriction!**

# The Model and Its Assumptions

- General model in "standard" form:

$$\max z = cx$$
$$\text{subject to } Ax \leq b$$
$$x \geq 0$$

Note that z is a convenience

Note that x can bounded above and below; we'll deal with it later

- Assumptions
  - **Proportionality:** contribution to objective and consumption of resources are proportional to value of **x**
  - **Additivity:** contribution, consumption of $\mathbf{x_i}$ independent of value of $\mathbf{x_k}$
  - **Divisibility: x** can take on continuous values within its bounds
  - **Certainty:** ***A*,*b*,** or ***c*** are not random
- Is this too restrictive to be useful?

# Optimization Taxonomy - By Assumptions

| | Proportionality | Additivity | Divisibility | Certainty |
|---|---|---|---|---|
| Linear Programming | X | X | X | X |
| Integer Programming | X | X | | X |
| Nonlinear Programming | | | | X |
| Stochastic Programming (common forms) | X | X | X | |

# Aside: Some Recommended Sources

- Overall optimization modeling
  - Schrage, *Optimization Modeling with LINDO*
  - H. P. Williams, *Model Building in Mathematical Programming*
- Linear, Integer Programming
  - Rardin, *Optimization*
  - Vanderbei, *Linear Programming - Foundations and Extensions*
  - A very good set of course notes from Dr. John Chinneck: http://www.sce.carleton.ca/faculty/chinneck/po.html
- Networks
  - Ahuja, Magnanti, Orlin, *Network Flows*
- Nonlinear Optimization
  - Bazarra, Sherali, Shetty*, Nonlinear Programming - Theory and Algorithms*

# Graphical Solution of LP's

- Note: this is a teaching aid, not a serious technique
  - Limited to 2 (or maybe 3) variables
  - Idea is to use graphs to reinforce concepts
  - Will blast through this rapidly
- Infamous Hillier and Lieberman Wyndor Glass problem
  - In their OR book since 1967
  - Has survived unchanged through 5 subsequent editions
  - Will presumably be taught to your grandchildren as well

# Wyndor Glass Info

- Company has 3 plants
  - Plant 1: aluminum frames and hardware (4 units capacity avail)
  - Plant 2: wood frames (12 units capacity avail)
  - Plant 3: glass, overall assembly (18 units capacity avail)
- They can make 2 new products (and sell all production)
  - 1: 8-ft glass door, aluminum framing ($3 profit/unit)
  - 2: 4x6 double-hung wood window ($5 profit/unit)
- Capacity required per unit by product at each plant

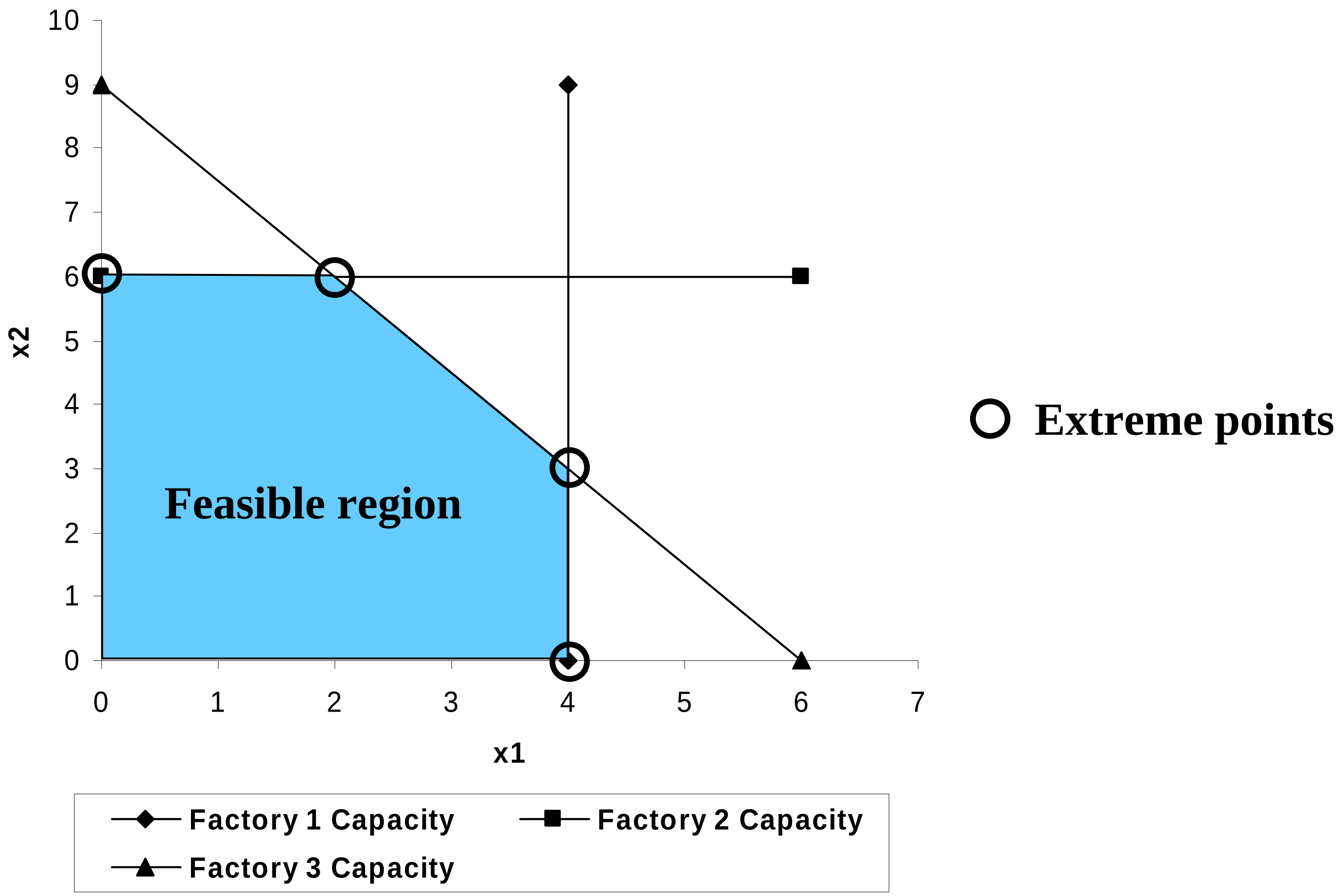
| | **Plant** | | |
|---|---|---|---|
| **Product** | **1** | **2** | **3** |
| **1** | 1 | 0 | 3 |
| **2** | 0 | 2 | 2 |

# Problem Formulation

- In words:
  - Maximize total profit
  - By choice of production amounts for the two new products
  - Subject to constraints on manufacturing capacity
- The Math:
  - define $\boldsymbol{x_i}$ as production of product $\boldsymbol{i}$; then the problem is:

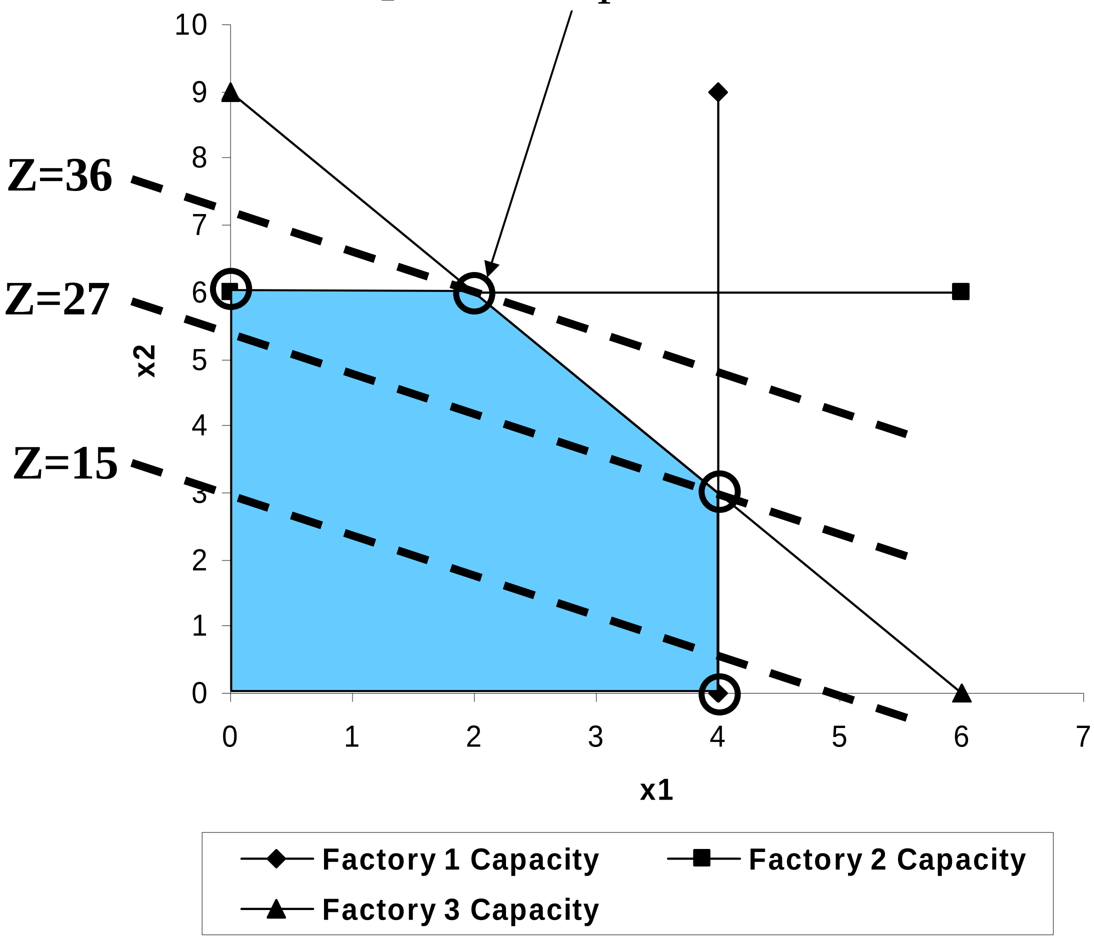
$$
\begin{aligned}
&\max \quad Z = 3x_1 + 5x_2 \\
&\text{subject to:} \\
&x_1 \le 4 \text{ (plant 1 capacity)} \\
&2x_2 \le 12 \text{ (plant 2 capacity)} \\
&3x_1 + 2x_2 \le 18 \text{ (plant 3 capacity)} \\
&x_1, x_2 \ge 0 \text{ (no negative production)}
\end{aligned}
$$

# Graphing the Constraints; Terminology



# Objective Function Contours

Optimum: $x_1$=2, x2=6



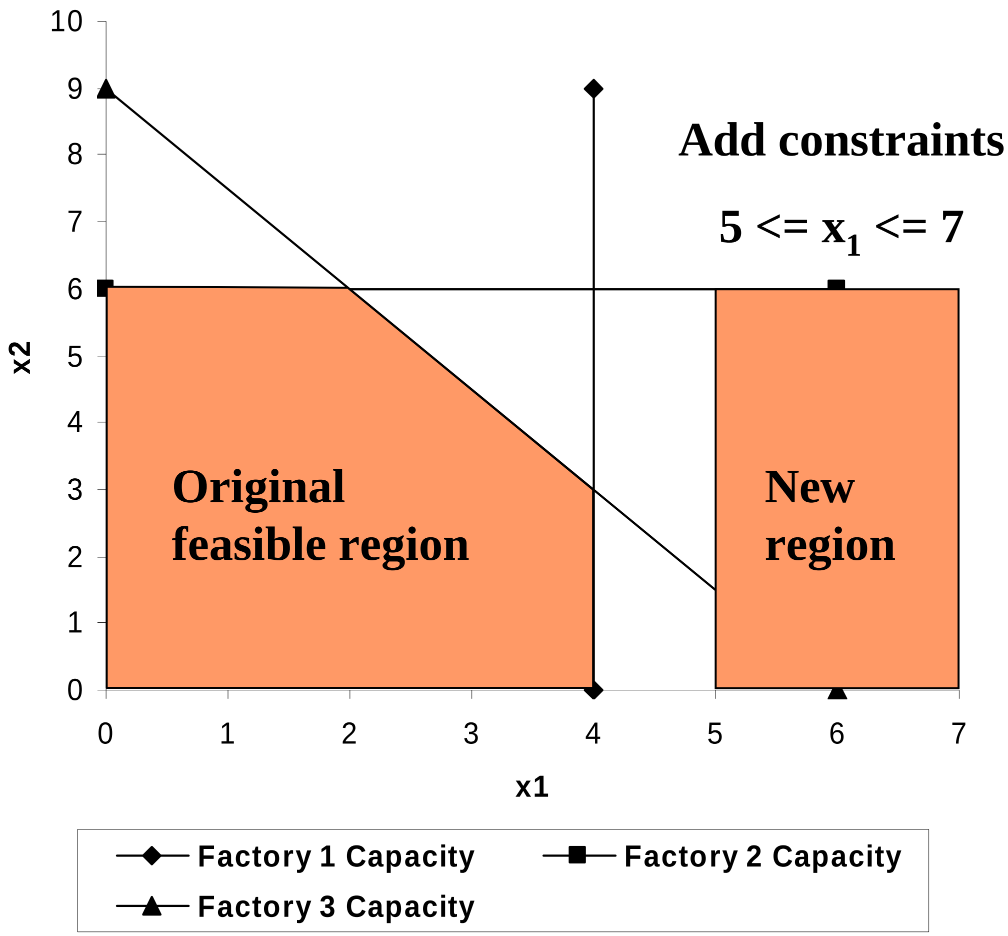

**Method:**

1. Set $Z = 3x_1 + 5x_2$ to some value
2. Plot contour
3. Find parallel contour that intersects feasible region and maximizes Z (greatest orthogonal distance from the point (0,0)

# Some Questions

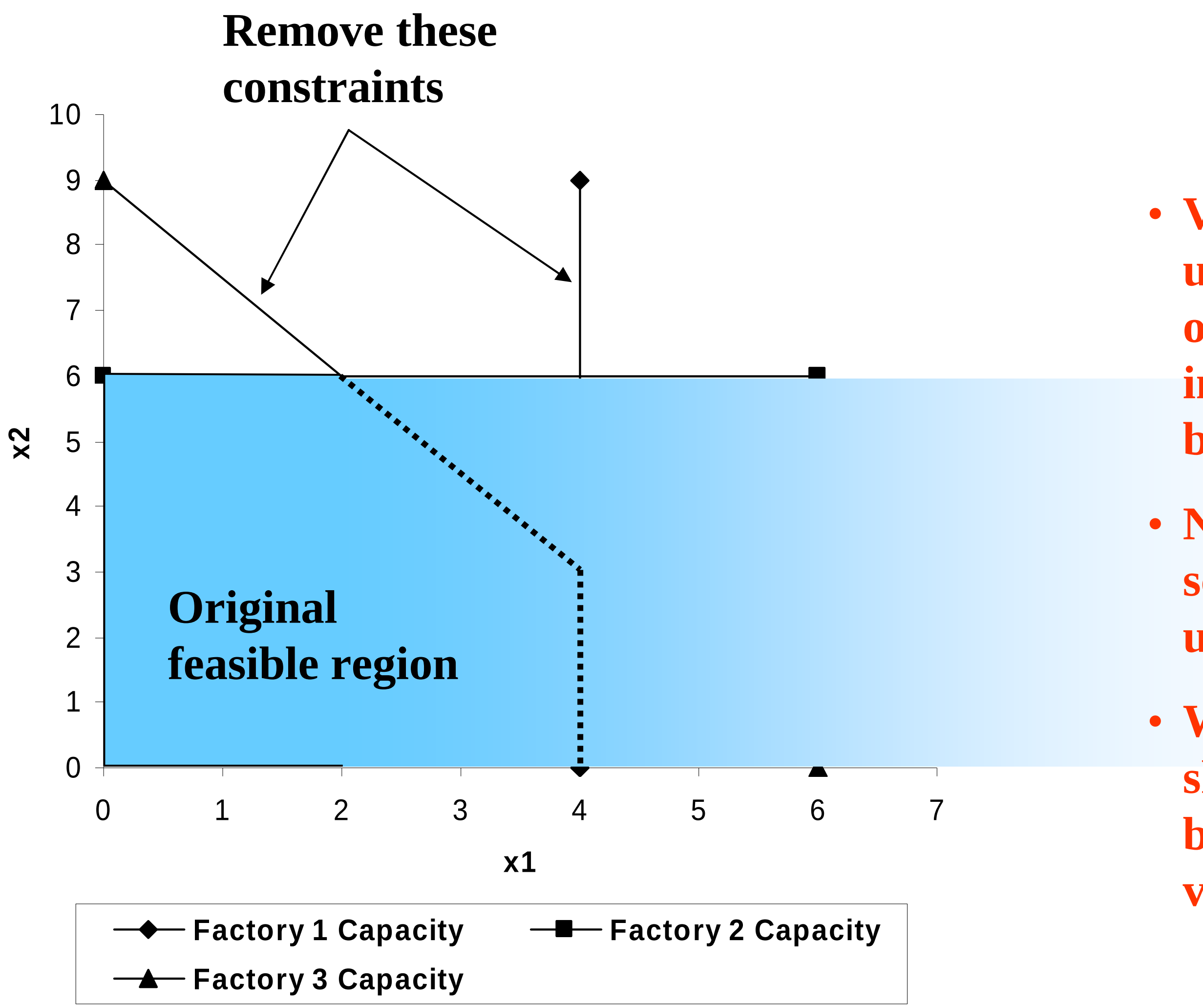
- What if the answer wasn't integral? (IP part of the course)
- Is the feasible region always a convex set?
  - Euclidean space $\mathbf{E}^n$: set of all n-dimensional vectors of real #'s
  - Convex set **S** in Euclidean space $\mathbf{E}^n$: any line segment joining two points in the set is also in the set, i.e., if $\mathbf{x}_1$ and $\mathbf{x}_2$ are in **S**, then $\mathbf{a}\mathbf{x}_1+(1-\mathbf{a})\mathbf{x}_2$ is also in **S**, for $\mathbf{0 <= a <= 1}$
  - Each constraint forms a *half-space*; intersection of a finite set of half-spaces is a *polyhedron,* which is a convex set
- Does the optimum always occur at an extreme point?
  - Extreme point of convex set **S**: a point which cannot be written as a *strict* convex combination of any 2 points in **S**, i.e., $\mathbf{x} <> \mathbf{a}\mathbf{x}_1+(1-\mathbf{a})\mathbf{x}_2$, for any $\mathbf{x}_1, \mathbf{x}_2$, and $\mathbf{0<a<1}$
  - Suppose a point **x** isn't an extreme point, but is optimum. What happens?

# Special Case #1 - Infeasible Problem



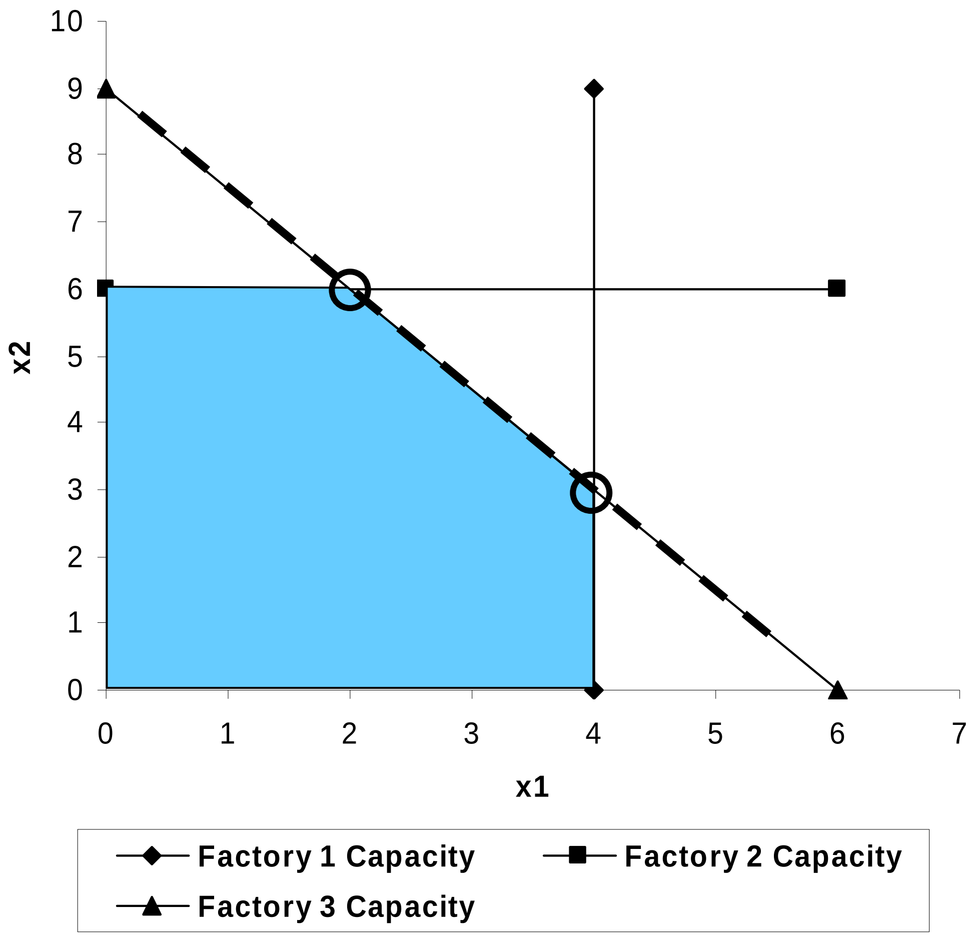

- No point satisfies constraints; problem's DOA
- NOTE: discovering which constraints are irreconcilable can be *very* difficult in large problems

# Special Case #2 - Unbounded Solution




- Variable x1 unconstrained; objective can increase without bound
- Normally sign of a screwup; nothing's unbounded in reality
- When modeling, you should put simple bounds on *all* variables

# Special Case #3: Infinite Number of Solutions




- **Common in large LPs**
- **Objective function is parallel to a binding constraint**
- **Note that the optimum still occurs at extreme points (but also on the line segment connecting them)**
- **Generally leads to more analysis to try to break ties**

# One More Special Case ...

- Frequently, a variable will appear in the solution with a value of 0 (or be at its upper or lower bound)
- This is a condition known as *degeneracy*
- Hard to show in two dimensions; we'll cover it later