Transportation Problem

x_{11} 8	x_{12} 6	x_{13} 10	x_{14} 9	35
x_{21} 9	12	13	x_{24} 7	50
x_{31} 14	9	16	x_{34} 5	40
45	20	30	30	

$$z = 8x_{11} + ... + 9x_{14}$$

$$+ .....$$

$$14x_{31} + ... + 5x_{34}$$

s.t.

$$x_{11} + x_{12} + x_{13} + x_{14} = 35$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 50$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 40$$

$$x_{11} + x_{21} + x_{31} = 45$$

$$x_{12} + x_{22} + x_{32} = 20$$

$$x_{13} + x_{23} + x_{33} = 30$$

$$x_{14} + x_{24} + x_{34} = 30$$

$$x_{ij} \geq 0$$

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} \le s_i, \quad i = 1,...,m \ (Supply \ \ constr.)$$

$$\sum_{i=1}^{m} x_{ij} \ge d_j, \quad j = 1,...,n \ (Demand \ \ constr.)$$

$$x_{ij} \ge 0$$

If $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$

then it is said to be balanced

Transportation problem

$$\sum s_i > \sum d_j \Rightarrow n+1 \text{ demand point}$$

$$d_{n+1} = \sum_{i=1}^{m} s_i - \sum_{j=1}^{n} d_j$$

$$c_{in+1} = 0$$

$$\sum s_i < \sum d_j \text{ then } (m+1) \text{ Supply point}$$

$$s_{m+1} = \sum d_j - \sum s_i$$

$c_{m+1,j} = M$ *for those demand points where the product has to be delivered*

M – penalty

				30
				50
				80
20	30	40	50	

$$\sum s_i - \sum d_j = 160 - 140 = 20$$

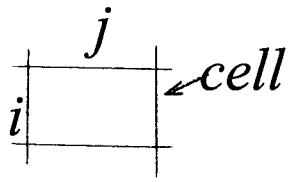
				0	30
				0	50
				0	80
20	30	40	50	20	

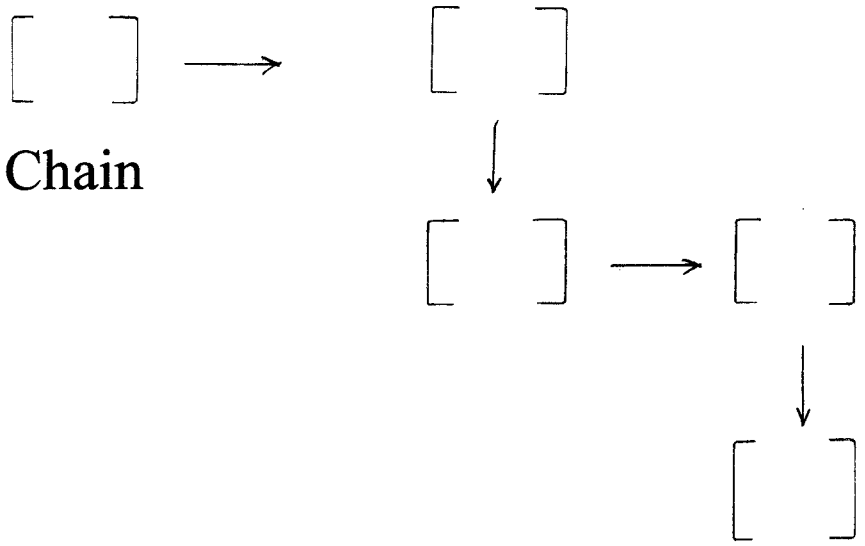
				30
				50
				80
100	50	30	40	

$$\Downarrow$$

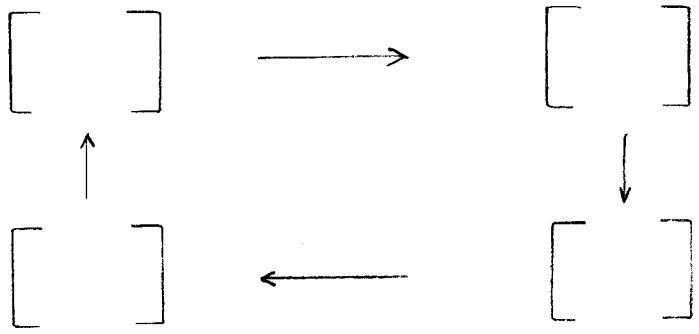
$$\sum s_i - \sum d_j = 160 - 220 = -60$$

				30
				50
				80
M				60
100	50	30	40	

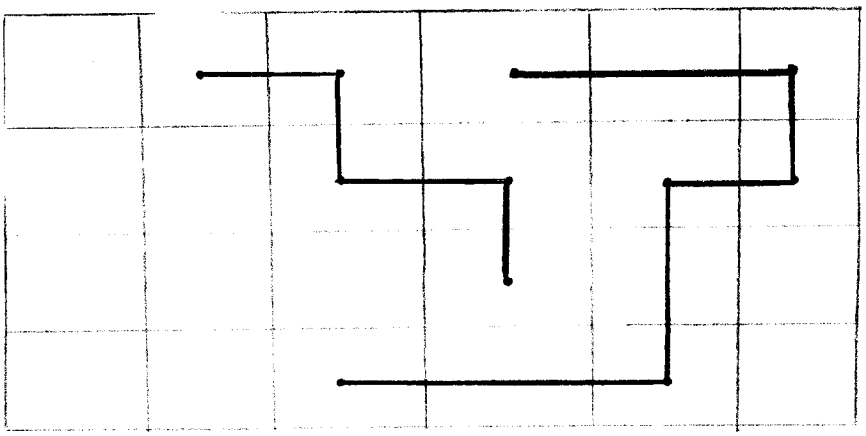




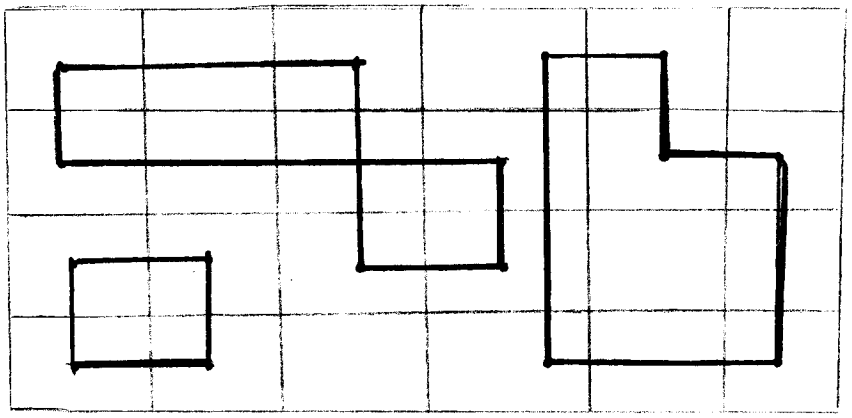
Chain



Closed chain – loop

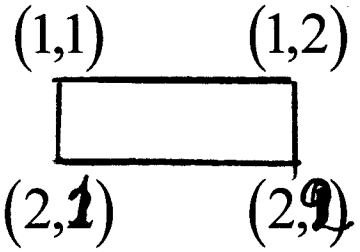


chain



Loop

x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}	
1	1	1	1									35
				1	1	1	1					50
								1	1	1	1	40
1				1				1				45
	1				1				1			20
		1				1				1		30
			1				1				1	30



$$\begin{pmatrix}1\\0\\0\\1\\0\\0\\0\end{pmatrix} - \begin{pmatrix}1\\0\\0\\0\\1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\0\\0\\1\\0\\0\end{pmatrix} - \begin{pmatrix}0\\1\\0\\1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\end{pmatrix}$$

Loop $\Leftrightarrow$ Linear dependent columns

m + n –1 Linear independent columns

m + n –1 cells without loop

North – West Corner

35	8	6	10	9
9		12	13	7
14		9	16	5
45	10	20	30	30

35
50
40

Min Element

Vogel's Method

35^{18}	x^{6}	x^{5}	x^{9}	35
10^{10}	20^{16}	20^{8}	^{9}	50
x^{3}	x^{11}	10^{6}	30^{14}	40
45	20	30	30	

$x_{ij} > 0 \quad m+n-1$

$$\begin{cases} v_j - u_i = c_{ij} & m+n-1 \ \textit{equations} \\ & m+n \ \ \text{var}\textit{iables} \end{cases}$$

$u_1 = 0$

$v_1 = 18$

$u_1 = 0$					35
$u_2 =$					50
					40
	45	20	30	30	

Dual Problems & optimality criteria

$$\min z = \sum\sum c_{ij} x_{ij}$$

$$-u_i \quad \sum_{j=1}^{n} x_{ij} = s_i \,, \quad i = 1,\ldots,m$$

$$v_j \quad \sum_{i=1}^{m} x_{ij} = d_j \,, \quad j = 1,\ldots,n$$

$$x_{ij} \geq 0$$

$$\max w = \sum_{j=1}^{n} v_j d_j - \sum_{i=1}^{m} u_i s_i$$

$$v_j - u_i \leq c_{ij}$$

Complementarity condition

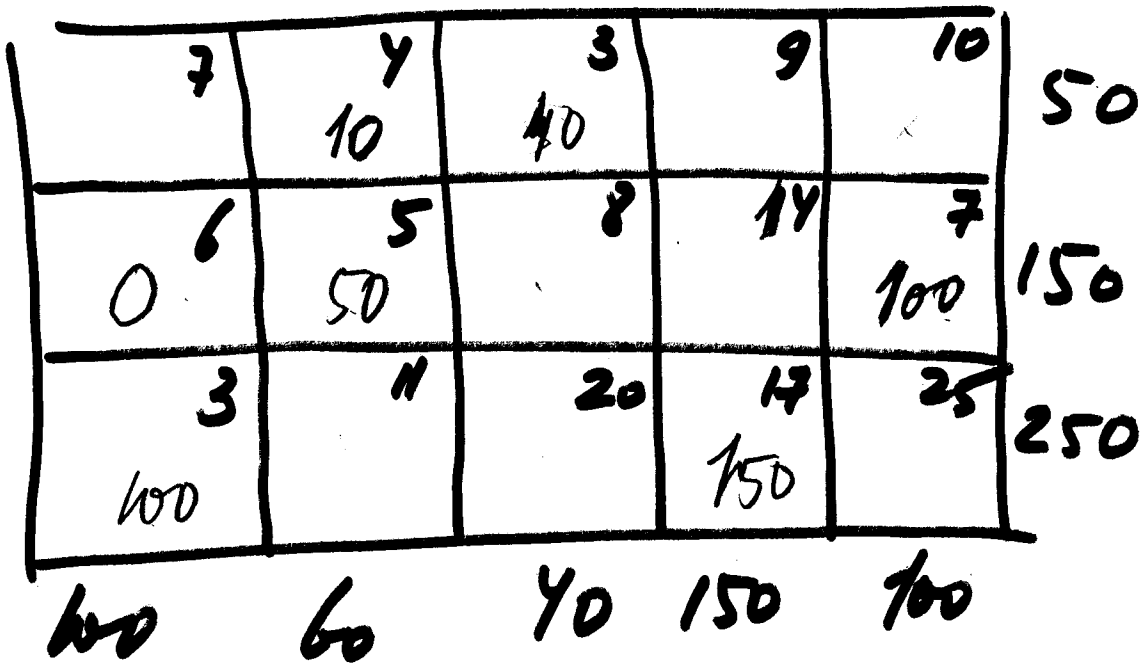
$$x_{ij} > 0 \Rightarrow v_j - u_i - c_{ij} = 0$$

$$x_{ij} = 0 \Leftarrow v_j - u_i - c_{ij} < 0$$

		City 1	City 2	City 3	City 4	Supply
	$v_j =$	6	6	10	2	
Plant 1	$u_i = 0$	8	6 10	10 25	9	35
Plant 2	3	9 45	12	13 5	7	50
Plant 3	3	14	9 10	16	5 30	40
Demand		45	20	30	30	

		City 1	City 2	City 3	City 4	Supply
	$v_j =$	6	6	10	2	
Plant 1	$u_i = 0$	8	6 12	10 25	9	37
Plant 2	3	9 45	12	13 5	7	50
Plant 3	3	14	9 10	16	5 30	40
Demand		45	22	30	30	

		City 1	City 2	City 3	City 4	Supply
	$v_j =$	6	6	10	2	
Plant 1	$u_i = 0$	8	6 10	10 26	9	36
Plant 2	3	9 46	12	13 4	7	50
Plant 3	3	14	9 10	16	5 30	40
Demand		46	20	30	30	


7
4
10
3
40
9
10
50
6
0
5
50
8
14
7
100
150
3
100
11
20
17
150
25
250
100
60
40
150
100

8 35	6 X	10 X	9 X	~~35~~ 0
9 10	12	13	7	~~50~~ 40
14 X	9	16	5	40
~~45~~ ~~40~~ 0	20	30	30	

~~8~~ ~~35~~	6 X	10 X	9 X	~~35~~ 0
9 10	12 20	13	7	~~50~~ ~~40~~ 20
14 X	9 X	16	5	40
~~45~~ ~~40~~ 0	~~20~~ 0	30	30	

Sensitivity Analysis for TP

Changing the OF coeficient of a NBV

	6	6	10	2	
$u_1 = 0$	^{8}	$10\ ^{6}$	$25\ ^{10}$	^{9}	35
-3	$45\ ^{9}$	^{12}	$5\ ^{13}$	^{7}	50
-3	^{14}	$10\ ^{9}$	^{16}	$30\ ^{5}$	40
	45	20	30	30	

a) $v_4 - u_1 \le c_{14} + \Delta \Rightarrow 2 \le 9 + \Delta\ ;\ \Delta \ge -7$

$2 \le c_{14} \le \infty$

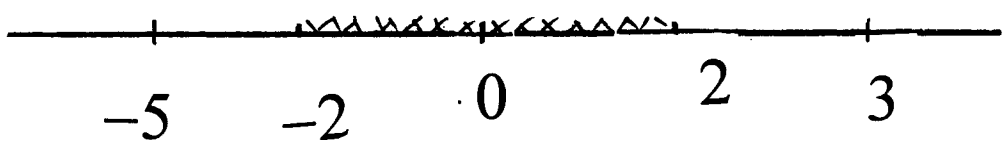
BV

b) $c_{13} + \Delta$

	$6-\Delta$	6	$10+\Delta$	2
0	^{8}	$10\ ^{6}$	$25\ ^{10+\Delta}$	^{9}
$-3-\Delta$	$45\ ^{9}$	^{12}	$5\ ^{13}$	^{7}
-3	^{14}	$10\ ^{9}$	^{16}	$30\ ^{5}$

$$6-\Delta-0\le 8,\quad 6-(-3-\Delta)\le 12$$
$$2-(-3-\Delta)\le 7,\quad 6-\Delta-(-3)\le 14$$
$$10+\Delta-(-3)\le 16$$

$$\Delta\ge -2,\quad 9+\Delta\le 12,\quad \Delta\le 3$$
$$5+\Delta\le 7,\quad \Delta\le 2,\quad 9-\Delta\le 14,\quad \Delta\ge -5$$
$$13+\Delta\le 16,\quad \Delta\le 3$$



Assignment Problem

$$\begin{matrix} 5 & 7 & 9 & 6 \\ 11 & 4 & 13 & 17 \\ 8 & 9 & 4 & 13 \\ 10 & 7 & 11 & 9 \end{matrix} \Rightarrow \begin{matrix} 0 & 2 & 4 & 1 \\ 7 & 0 & 9 & 13 \\ 4 & 5 & 0 & 9 \\ 3 & 0 & 4 & 2 \end{matrix}$$

$$\Downarrow$$

$$\begin{matrix} 0^{*} & 2 & 4 & 0 \\ 7 & 0^{*} & 9 & 12 \\ 4 & 5 & 0^{*} & 8 \\ 3 & 0 & 4 & 1 \end{matrix}$$

$$\begin{array}{cccc} 0 & 2 & 4 & 0 \\ 7 & 0 & 9 & 12 \\ 4 & 5 & 0 & 8 \\ 3 & 0 & 4 & 1 \\ & +1 & & \end{array} \qquad \begin{array}{c} \\ -1 \\ \\ -1 \\ \end{array}$$

$\min\{7,9,12,3,4,1\} = 1 = k$

Subtract $k > 0$ *from each uncovered row and add* $k > 0$ *to covered columns*

$$\begin{array}{cccc} 0^{*} & 3 & 4 & 0 \\ 6 & 0^{*} & 8 & 11 \\ 4 & 6 & 0^{*} & 8 \\ 2 & 0 & 3 & 0^{*} \end{array}$$

Subtract k from each uncovered element and add k to each element which is covered twice

17	5	8	7
2	12	6	5
7	8	3	9
2	4	6	10

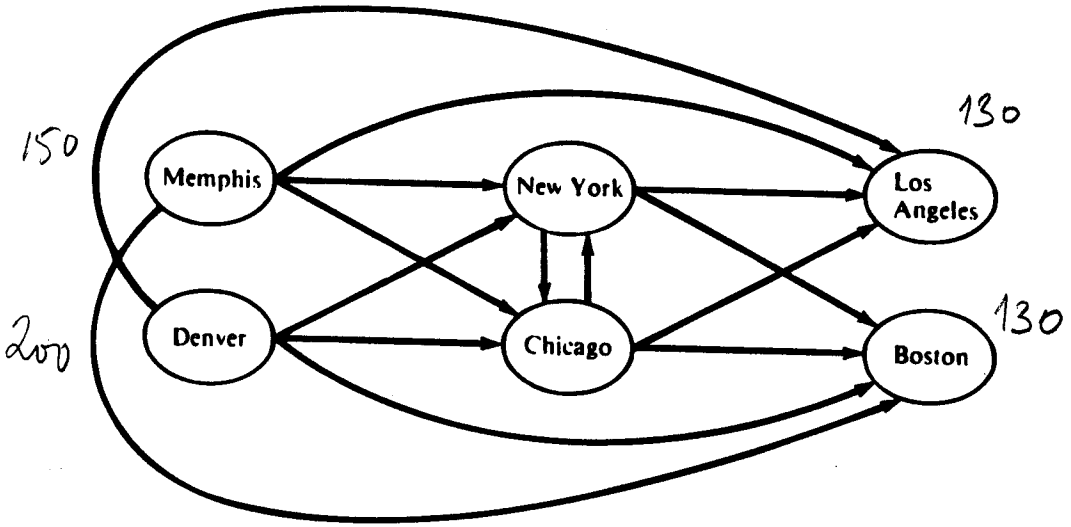
12	0	3	2
0	10	4	3
4	5	0	6
0	2	4	8

$$
\begin{array}{cccc}
19 & 0 & 3 & 0 \\
0 & 10 & 4 & 1 \quad -1 \\
4 & 5 & 0 & 4 \\
0 & 2 & 4 & 6 \quad -1
\end{array}
$$

$+1$

$$
\begin{array}{cccc}
18 & 0 & 3 & 0 \\
0 & 9 & 3 & 0 \\
5 & 5 & 0 & 4 \\
0 & 1 & 3 & 5
\end{array}
$$

From	To					
	Memphis	Denver	N.Y.	Chicago	L.A.	Boston
Memphis	$0	—	$8	$13	$25	$28
Denver	—	$0	$15	$12	$26	$25
N.Y.	—	—	$0	$6	$16	$17
Chicago	—	—	$6	$0	$14	$16
L.A.	—	—	—	—	$0	—
Boston	—	—	—	—	—	$0



	NY	Ch	LA	Bos	D	
Mem	8	13	25	28	0	150
Den	15	12	26	25	0	200
NY	0	6	16	17	M	260
Ch	6	0	14	16	M	260
	260	260	130	130	90	