Use the LINDO output in Figure 6 to answer the following questions:

a If only 40 acres of land were available, what would Leary's profit be?

b If the price of wheat dropped to $26, what would be the new optimal solution to Leary's problem?

c Use the SLACK portion of the output to determine the allowable increase and allowable decrease for the amount of wheat that can be sold. If only 130 bushels of wheat could be sold, then would the answer to the problem change?

2 Carco manufactures cars and trucks. Each car contributes $300 to profit, and each truck contributes $400. The resources required to manufacture a car and a truck are shown in Table 5. Each day, Carco can rent up to 98 Type 1 machines at a cost of $50 per machine. The company has 73 Type 2 machines and 260 tons of steel available. Marketing considerations dictate that at least 88 cars and at least 26 trucks be produced. Let x_1 = number of cars produced daily; x_2 = number of trucks produced daily; and m_1 = Type 1 machines rented daily.

To maximize profit, Carco should solve the LP in Figure 7. Use the LINDO output to answer the following questions:

a If each car contributed $310 to profit, what would be the new optimal solution to the problem?

TABLE 5

Vehicle	Days on Type 1 Machine	Days on Type 2 Machine	Tons of Steel
Car	0.8	0.6	2
Truck	1	0.7	3

FIGURE 6

LINDO Output for Wheat and Corn

```
MAX     150  A1 + 200  A2  -  10 L
SUBJECT TO
        2)     A1 + A2 <=   45
        3)     6 A1 + 10 A2 - L <=  0
        4)     L <=   350
        5)     5 A1 <=   140
        6)     4 A2 <=   120
END

LP OPTIMUM FOUND AT STEP          4

                 OBJECTIVE FUNCTION VALUE

                 1)   4250.00000

VARIABLE         VALUE          REDUCED COST
      A1        25.000000           .000000
      A2        20.000000           .000000
       L       350.000000           .000000

     ROW   SLACK OR SURPLUS     DUAL PRICES
      2)          .000000       75.000000
      3)          .000000       12.500000
      4)          .000000        2.500000
      5)        15.000000         .000000
      6)        40.000000         .000000

NO. ITERATIONS=          4

RANGES IN WHICH THE BASIS IS UNCHANGED:

                   OBJ COEFFICIENT RANGES
VARIABLE  CURRENT      ALLOWABLE       ALLOWABLE
            COEF       INCREASE        DECREASE
      A1  150.000000  10.000000       30.000000
      A2  200.000000  50.000000       10.000000
       L  -10.000000   INFINITY        2.500000

                   RIGHTHAND SIDE RANGES
     ROW  CURRENT      ALLOWABLE       ALLOWABLE
            RHS        INCREASE        DECREASE
       2   45.000000    1.200000        6.666667
       3     .000000   40.000000       12.000000
       4  350.000000   40.000000       12.000000
       5  140.000000    INFINITY       15.000000
       6  120.000000    INFINITY       40.000000
```

FIGURE 7

LINDO Output for Carco

```
MAX      300  X1 + 400  X2  -  50  M1
SUBJECT TO
         2)     0.8 X1  +  X2  -  M1  <=   0
         3)     M1  <=    98
         4)     0.6  X1 +  0.7  X2 <=    73
         5)     2 X1   + 3  X2   <=   260
         6)     X1 >=  88
         7)     X2 >=  26
END

LP OPTIMUM FOUND AT STEP                  4

                 OBJECTIVE FUNCTION VALUE

                 1)   32540.0000

VARIABLE         VALUE          REDUCED COST
      X1        88.000000          .000000
      X2        27.600000          .000000
      M1        98.000000          .000000

     ROW   SLACK OR SURPLUS     DUAL PRICES
      2)          .000000      400.000000
      3)          .000000      350.000000
      4)          .879999         .000000
      5)         1.200003         .000000
      6)          .000000      -20.000000
      7)         1.599999         .000000

NO. ITERATIONS=          4

RANGES IN WHICH THE BASIS IS UNCHANGED:

                   OBJ COEFFICIENT RANGES
VARIABLE   CURRENT      ALLOWABLE      ALLOWABLE
             COEF       INCREASE       DECREASE
      X1   300.000000   20.000000      INFINITY
      X2   400.000000    INFINITY     25.000000
      M1   -50.000000    INFINITY    350.000000

                   RIGHTHAND SIDE RANGES
     ROW   CURRENT       ALLOWABLE      ALLOWABLE
             RHS         INCREASE       DECREASE
       2     .000000      .400001       1.599999
       3   98.000000      .400001       1.599999
       4   73.000000     INFINITY        .879999
       5  260.000000     INFINITY       1.200003
       6   88.000000     1.999999       3.000008
       7   26.000000     1.599999      INFINITY
```

TABLE 8

Steel	Coal Required (Tons)	Iron Required (Tons)	Labor Required (Hours)	Sales Price ($)
1	3	1	1	51
2	2	0	1	30
3	1	1	1	25

FIGURE 11
LINDO Output for Steelco

```
MAX         8  X1 + 5  X2  + 2  X3
SUBJECT TO
       2)      3  X1  +  2  X2  +  X3 <=  200
       3)      X1  +  X3    <=      60
       4)      X1  +  X2  +  X3 <=  100
END

LP OPTIMUM FOUND AT STEP            2

        OBJECTIVE FUNCTION VALUE

        1)   530.000000

VARIABLE          VALUE          REDUCED COST
      X1        60.000000           .000000
      X2        10.000000           .000000
      X3          .000000          1.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)          .000000          2.500000
       3)          .000000           .500000
       4)        30.000000           .000000

NO. ITERATIONS=          2

RANGES IN WHICH THE BASIS IS UNCHANGED:

                          OBJ COEFFICIENT RANGES
VARIABLE       CURRENT        ALLOWABLE         ALLOWABLE
                 COEF          INCREASE          DECREASE
      X1       8.000000        INFINITY           .500000
      X2       5.000000         .333333          5.000000
      X3       2.000000        1.000000          INFINITY

                          RIGHTHAND SIDE RANGES
     ROW       CURRENT        ALLOWABLE         ALLOWABLE
                 RHS           INCREASE          DECREASE
       2     200.000000       60.000000         20.000000
       3      60.000000        6.666667         60.000000
       4     100.000000        INFINITY         30.000000
```

b What is the smallest price per ton for steel 3 that would make it desirable to produce it?

c Find the new optimal solution if steel 1 sold for $55 per ton.

Group B

7 Shoeco must meet (on time) the following demands for pairs of shoes: month 1—300; month 2—500; month 3—100; and month 4—100. At the beginning of month 1, 50 pairs of shoes are on hand, and Shoeco has three workers. A worker is paid $1,500 per month. Each worker can work up to 160 hours per month before receiving overtime. During any month, each worker may be forced to work up to 20 hours of overtime; workers are paid $25 per hour for overtime labor. It takes 4 hours of labor and $5 of raw material to produce each pair of shoes. At the beginning of each month, workers can be hired or fired. Each hired worker costs $1,600, and each fired worker costs $2,000. At the end of each month, a holding cost of $30 per pair of shoes is assessed. Formulate an LP that can be used to minimize the total cost of meeting the next four months' demands. Then use LINDO to solve the LP. Finally, use the LINDO printout to answer the questions that follow these hints (which may help in the formulation.) Let

x_t = Pairs of shoes produced during month t with nonovertime labor

o_t = Pairs of shoes produced during month t with overtime labor

i_t = Inventory of pairs of shoes at end of month t

h_t = Workers hired at beginning of month t

f_t = Workers fired at beginning of month t

w_t = Workers available for month t (after month t hiring and firing)

Four types of constraints will be needed:

Type 1 Inventory equations. For example, during month 1, $i_1 = 50 + x_1 + o_1 - 300$.

Type 2 Relate available workers to hiring and firing. For month 1, for example, the following constraint is needed: $w_1 = 3 + h_1 - f_1$.

Type 3 For each month, the amount of shoes made with nonovertime labor is limited by the number of workers. For example, for month 1, the following constraint is needed: $4x_1 \leq 160w_1$.

Type 4 For each month, the number of overtime labor hours used is limited by the number of workers. For example, for month 1, the following constraint is needed: $4(o_1) \leq 20w_1$.

For the objective function, the following costs must be considered:

1 Workers' salaries
2 Hiring costs
3 Firing costs
4 Holding costs
5 Overtime costs
6 Raw-material costs

a Describe the company's optimal production plan, hiring policy, and firing policy. Assume that it is acceptable to have a fractional number of workers, hirings, or firings.

b If overtime labor during month 1 costs $16 per hour, should any overtime labor be used?

c If the cost of firing workers during month 3 were $1,800, what would be the new optimal solution to the problem?

d If the cost of hiring workers during month 1 were $1,700, what would be the new optimal solution to the problem?

e By how much would total costs be reduced if demand in month 1 were 100 pairs of shoes?

f What would the total cost become if the company had 5 workers at the beginning of month 1 (before month 1's hiring or firing takes place)?

FIGURE 9
LINDO Output for Gepbab

```
MAX      5  X11 + 6 X12 + 8 X13 + 8 X21
                            + 7 X22 + 10 X23
SUBJECT TO
        2)     X11  +  X12  +  X13 <=   10000
        3)     X21  +  X22  +  X23 <=   10000
        4)     X11  +  X21  >=     6000
        5)     X12  +  X22  >=     8000
        6)     X13  +  X23  >=     5000
END

LP OPTIMUM FOUND AT STEP            5

                  OBJECTIVE FUNCTION VALUE

                  1)   128000.000

VARIABLE        VALUE              REDUCED COST
     X11     6000.000000              .000000
     X12         .000000             1.000000
     X13     4000.000000              .000000
     X21         .000000             1.000000
     X22     8000.000000              .000000
     X23     1000.000000              .000000


     ROW    SLACK OR SURPLUS     DUAL PRICES
      2)         .000000          2.000000
      3)     1000.000000           .000000
      4)         .000000         -7.000000
      5)         .000000         -7.000000
      6)         .000000        -10.000000

NO. ITERATIONS=          5

RANGES IN WHICH THE BASIS IS UNCHANGED:

                          OBJ COEFFICIENT RANGES
VARIABLE      CURRENT            ALLOWABLE           ALLOWABLE
               COEF              INCREASE            DECREASE
     X11   5.000000              1.000000            7.000000
     X12   6.000000              INFINITY            1.000000
     X13   8.000000              1.000000            1.000000
     X21   8.000000              INFINITY            1.000000
     X22   7.000000              1.000000            7.000000
     X23  10.000000              1.000000            1.000000

                          RIGHTHAND SIDE RANGES
     ROW      CURRENT            ALLOWABLE           ALLOWABLE
                RHS              INCREASE            DECREASE
       2   10000.000000       1000.000000         1000.000000
       3   10000.000000          INFINITY         1000.000000
       4    6000.000000       1000.000000         1000.000000
       5    8000.000000       1000.000000         8000.000000
       6    5000.000000       1000.000000         1000.000000
```

b How much money would Mondo save if the capacity of plant 3 were increased by 100 motorcycles?

c By how much would Mondo's cost increase if it had to produce one more motorcycle?

6 Steelco uses coal, iron, and labor to produce three types of steel. The inputs (and sales price) for one ton of each type of steel are shown in Table 8. Up to 200 tons of coal can be purchased at a price of $10 per ton. Up to 60 tons of iron can be purchased at $8 per ton, and up to 100 labor hours can be purchased at $5 per hour. Let x_1 = tons of steel 1 produced; x_2 = tons of steel 2 produced; and x_3 = tons of steel 3 produced.

TABLE 7

Plant	Labor Needed (Hours)	Raw Material Needed (Units)	Production Cost ($)
1	20	5	50
2	16	8	80
3	10	7	100

FIGURE 10
LINDO Output for Mondo

```
MAX        300  X1 + 280  X2  +  225  X3
SUBJECT TO
         2)     20 X1  + 16 X2  + 10  X3 <=  21000
         3)     5  X1  + 8  X2  +  7  X3 <=  9400
         4)     X1  <=      750
         5)     X2  <=      750
         6)     X3  <=      750
         7)     X1  +  X2  +  X3  >=    1400
END

LP OPTIMUM FOUND AT STEP               3

                  OBJECTIVE FUNCTION VALUE

                  1)   357750.000

VARIABLE        VALUE          REDUCED COST
      X1     350.000000           .000000
      X2     300.000000           .000000
      X3     750.000000           .000000


     ROW   SLACK OR SURPLUS   DUAL PRICES
      2)     1700.000000         .000000
      3)         .000000        6.666668
      4)      400.000000         .000000
      5)      450.000000         .000000
      6)         .000000       61.666660
      7)         .000000     -333.333300

NO. ITERATIONS=          3

RANGES IN WHICH THE BASIS IS UNCHANGED:

                        OBJ COEFFICIENT RANGES
VARIABLE    CURRENT          ALLOWABLE          ALLOWABLE
             COEF            INCREASE           DECREASE
      X1   300.000000        INFINITY          20.000000
      X2   280.000000       20.000010          92.499990
      X3   225.000000       61.666660           INFINITY

                        RIGHTHAND SIDE RANGES
     ROW    CURRENT          ALLOWABLE          ALLOWABLE
              RHS            INCREASE           DECREASE
       2  21000.000000       INFINITY         1700.000000
       3   9400.000000    1050.000000          900.000000
       4    750.000000       INFINITY          400.000000
       5    750.000000       INFINITY          450.000000
       6    750.000000     450.000000          231.818200
       7   1400.000000      63.750000          131.250000
```

The LINDO output that yields a maximum profit for the company is given in Figure 11. Use the output to answer the following questions.

a What would profit be if only 40 tons of iron could be purchased?

b If Carco were required to produce at least 86 cars, what would Carco's profit become?

3 Consider the diet problem discussed in Section 3.4. Use the LINDO output in Figure 8 to answer the following questions.

a If a Brownie costs 30¢, then what would be the new optimal solution to the problem?

b If a bottle of cola cost 35¢, then what would be the new optimal solution to the problem?

c If at least 8 oz of chocolate were required, then what would be the cost of the optimal diet?

d If at least 600 calories were required, then what would be the cost of the optimal diet?

e If at least 9 oz of sugar were required, then what would be the cost of the optimal diet?

f What would the price of pineapple cheesecake have to be before it would be optimal to eat cheesecake?

g What would the price of a brownie have to be before it would be optimal to eat a brownie?

h Use the SLACK or SURPLUS portion of the LINDO output to determine the allowable increase and allowable decrease for the fat constraint. If 10 oz of fat were required, then would the optimal solution to the problem change?

FIGURE 8
LINDO Output for Diet Problem

```
MAX      50 BR + 20 IC + 30 COLA + 80 PC
SUBJECT TO
       2)     400 BR + 200 IC + 150 COLA
                              + 500 PC >=   500
       3)     3  BR   + 2   IC   >=   6
       4)     2  BR   + 2   IC   +   4 COLA
                              + 4   PC >=   10
       5)     2  BR   + 4   IC   +   COLA
                              + 5   PC   >=   8
END

LP OPTIMUM FOUND AT STEP              2

                   OBJECTIVE FUNCTION VALUE

                   1)    90.0000000

VARIABLE         VALUE           REDUCED COST
      BR          .000000            27.500000
      IC         3.000000              .000000
    COLA         1.000000              .000000
      PC          .000000            50.000000


     ROW    SLACK OR SURPLUS     DUAL PRICES
      2)      250.000000            .000000
      3)         .000000          -2.500000
      4)         .000000          -7.500000
      5)        5.000000            .000000

NO. ITERATIONS=          2

RANGES IN WHICH THE BASIS IS UNCHANGED:

                       OBJ COEFFICIENT RANGES
VARIABLE      CURRENT       ALLOWABLE          ALLOWABLE
                COEF         INCREASE           DECREASE
      BR    50.000000        INFINITY          27.500000
      IC    20.000000       18.333330           5.000000
    COLA    30.000000       10.000000          30.000000
      PC    80.000000        INFINITY          50.000000

                       RIGHTHAND SIDE RANGES
     ROW      CURRENT       ALLOWABLE          ALLOWABLE
                RHS          INCREASE           DECREASE
       2   500.000000      250.000000           INFINITY
       3     6.000000        4.000000           2.857143
       4    10.000000        INFINITY           4.000000
       5     8.000000        5.000000           INFINITY
```

4 Gepbab Corporation produces three products at two different plants. The cost of producing a unit at each plant is shown in Table 6. Each plant can produce a total of 10,000 units. At least 6,000 units of product 1, at least 8,000 units of product 2, and at least 5,000 units of product 3 must be produced. To minimize the cost of meeting these demands, the following LP should be solved:

$$\begin{aligned}
\min z = 5x_{11} + 6x_{12} + 8x_{13} &+ 8x_{21} + 7x_{22} + 10x_{23} \\
\text{s.t.} \quad x_{11} + x_{12} + x_{13} & \le 10{,}000 \\
x_{21} + x_{22} + x_{23} & \le 10{,}000 \\
x_{11} + x_{21} & \ge 6{,}000 \\
x_{12} + x_{22} & \ge 8{,}000 \\
x_{13} + x_{23} & \ge 5{,}000 \\
\text{All variables} & \ge 0
\end{aligned}$$

Here, x_{ij} = number of units of product j produced at plant i. Use the LINDO output in Figure 9 to answer the following questions:

a What would the cost of producing product 2 at plant 1 have to be for the firm to make this choice?

b What would total cost be if plant 1 had 9,000 units of capacity?

c If it cost \$9 to produce a unit of product 3 at plant 1, then what would be the new optimal solution?

5 Mondo produces motorcycles at three plants. At each plant, the labor, raw material, and production costs (excluding labor cost) required to build a motorcycle are as shown in Table 7. Each plant has sufficient machine capacity to produce up to 750 motorcycles per week. Each of Mondo's workers can work up to 40 hours per week and is paid \$12.50 per hour worked. Mondo has a total of 525 workers and now owns 9,400 units of raw material. Each week, at least 1,400 Mondos must be produced. Let x_1 = motorcycles produced at plant 1; x_2 = motorcycles produced at plant 2; and x_3 = motorcycles produced at plant 3.

The LINDO output in Figure 10 enables Mondo to minimize the variable cost (labor + production) of meeting demand. Use the output to answer the following questions:

a What would be the new optimal solution to the problem if the production cost at plant 1 were only \$40?

TABLE 6

	Product ($)		
Plant	1	2	3
1	5	6	8
2	8	7	10

$$z = 3x_1 + 2x_2 + 3x_3 + 4x_4 \to \max$$

$$2x_1 + x_2 + 4x_3 + 2x_4 = 100$$

$$x_1 + x_2 + 2x_3 + 4x_4 = 80$$

$$x_i \geq 0$$

$$B = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

X= (20, 60, 0, 0)

$$B^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

1) $$B^{-1}\begin{pmatrix} 100+\Delta \\ 80 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 100+\Delta \\ 80 \end{pmatrix}$$

$$= 100 + \Delta - 80 \geq 0 \qquad \Delta \geq -20$$

$$-100 - \Delta + 160 \geq 0 \qquad \Delta \leq 60$$

$$80 \leq b_1 \leq 160$$

$$2)\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 100 \\ 80+\Delta \end{pmatrix} = \begin{cases} 100-80-\Delta \geq 0 \\ -100+160+2\Delta \geq 0 \end{cases}$$

$$-30 \leq \Delta \leq 20$$
$$50 \leq b_2 \leq 100$$

$$y = c_B B^{-1} = (3 \quad 2)\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$
$$= (1, 1)$$

$$3)\ \Delta_3 = (1, 1)\begin{pmatrix} 4 \\ 2 \end{pmatrix} - 3 = 3$$

$$-\infty < \Delta c_3 \leq \Delta_3 = 3 \qquad\qquad -\infty < c_3 \leq 6$$

$$\Delta_4 = (1,\ 1)\begin{pmatrix} 2 \\ 4 \end{pmatrix} - 4 = 2$$

$$-\infty < \Delta c_4 \le \Delta_4 = 2$$
$$-\infty < c_4 \le 6$$

$$4)\ c_1 + \Delta,\quad \hat{y} = (3+\Delta,\ 2)\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 3+\Delta-2 \\ 1-\Delta \end{pmatrix}$$

$$\hat{y} = (1+\Delta,\ 1-\Delta) \qquad \hat{y} \ge 0 \Rightarrow 1+\Delta \ge 0,\ \Delta \ge -1$$
$$1-\Delta \ge 0,\ \Delta \le 1$$

$$\Delta_3 = \left(\hat{y},\ A_3\right) - c_3 \ge 0 \qquad 4(1+\Delta) + (1-\Delta)2 - 3$$
$$= 4 + 4\Delta + 2 - 2\Delta - 3$$
$$= 3 + 2\Delta \ge 0,$$
$$\Delta \ge \frac{-3}{2}$$

$$\Delta_4 = \left(\hat{y} \quad A_4\right) - c_4 \geq 0$$

$$2(1+\Delta)+4(1-\Delta)-4$$
$$= 2+4+2\Delta-4\Delta-4 \geq 0$$

$$2 \geq 2\Delta, \qquad \Delta \leq 1$$

$$-1 \leq \Delta \leq 1$$
$$2 \leq c_1 \leq 4$$

$$\hat{y} = (3, \quad 2+\Delta)\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$= (3-2-\Delta, \quad -3+4+2\Delta) = (1-\Delta, \; 1+2\Delta)$$

$$\Delta_3 = 4(1-\Delta) + 2(1+2\Delta) - 3 \ge 0 \qquad \hat{y} \ge 0$$

$$4 - 4\Delta + 2 + 4\Delta - 3 \ge 0 \qquad \Downarrow$$

$$1-\Delta \ge 0 \quad \Delta \le 1$$

$$1+2\Delta \ge 0 \quad \Delta \ge \frac{-1}{2}$$

$$\Delta_4 = 2(1-\Delta) + 4(1+2\Delta) - 4 \ge 0$$

$$2 - 2\Delta + 4 + 8\Delta - 4 \ge 0$$

$$6\Delta \ge -2 \qquad \Delta \ge \frac{-1}{3}$$

$$\frac{-1}{3} \le \Delta \le 1 \qquad 1\frac{2}{3} \le c_2 \le 3$$

$$z^*(b) = c_B^T x_B = c_B^T B^{-1} b = (y, b)$$

$$z^*(b + \Delta b_i) = c_B^T x_{B+\Delta b_i} = c_B^T B^{-1}(b + \Delta b_i)$$
$$= (y, b + \Delta b_i)$$

$$z^*(b + \Delta b_i) - z^*(b) = (y, b + \Delta b_i) - (y, b)$$
$$= y_i \Delta b_i$$

$$y_i = \frac{z^*(b + \Delta b_i) - z^*(b)}{\Delta b_i}$$

$$3x_1 + 9x_2 + x_3 + 2x_4 \to \max$$
$$2x_1 + x_2 + 2x_3 + x_4 = 7$$
$$-x_1 + x_2 + 3x_3 + 2x_4 = 4$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0$$

B^{-1}

$$B = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \Rightarrow B^{-1} = ?$$

$X = (1, 5, 0, 0)$

$$\begin{pmatrix} 2 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ -1 & 1 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{3}{2} & \frac{1}{2} & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{-1}{3} \\ 0 & \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & \frac{1}{3} & \frac{-1}{3} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

$$\Rightarrow \quad B^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

$$B^{-1}(b) = \begin{pmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$b + \Delta b$:

$$\begin{pmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} 7+\Delta \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{7+\Delta}{3} - \frac{4}{3} \\ \frac{7+\Delta}{3} + \frac{8}{3} \end{pmatrix} \geq 0$$

$$1 + \frac{\Delta}{3} \geq 0 \qquad 3 + \Delta \geq 0, \qquad \Delta \geq -3$$

$$5 + \frac{\Delta}{3} \geq 0 \qquad 15 + \Delta \geq 0, \qquad \Delta \geq -15$$

$$-3 < \Delta_1 < +\infty$$

$$\begin{pmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} 7 \\ 4+\Delta \end{pmatrix} = \begin{pmatrix} \frac{7}{3} - \frac{4}{3} - \frac{\Delta}{3} \\ \frac{7}{3} + \frac{8}{3} + \frac{2\Delta}{3} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\Delta}{3} \\ 5 + \frac{2\Delta}{3} \end{pmatrix}$$

$$1 - \frac{\Delta}{3} \geq 0 \quad \Rightarrow \quad \Delta \leq 3$$

$$5 + \frac{2\Delta}{3} \geq 0 \quad \Rightarrow \quad \Delta \geq \frac{-15}{2}$$

$$\frac{-15}{2} \leq \Delta_2 \leq 3 \qquad -3{,}5 \leq b_2 \leq 7$$

	c_B			c_N	
	B			N	
	1	..	0		
c_B				$B^{-1}A_j$	$B^{-1}b$
	0	..	1		
Δ	0	..	0	Δ_j	

$$\Delta_j = c_B B^{-1} A_j - c_j = yA_j - c_j \geq 0$$

$$a)\ c_N: c_j \Rightarrow c_j + \Delta c_j$$

$$yA_j - \left(c_j + \Delta c_j\right) \geq 0$$

$$0 \leq \Delta c_j \leq yA_j - c_j = \Delta_j$$

$$y = c_B B^{-1}$$

$$y = \left(3, \quad 9\right)\begin{pmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{4}{3} & \frac{2}{3} \end{pmatrix} = \left(4, \quad 5\right)$$

$$\Delta_3 = \left(4, \quad 5\right)\begin{pmatrix} 2 \\ 3 \end{pmatrix} - 1 = 22$$

$$0 \leq \Delta c_3 \leq 22$$

$$\Delta_4 = \left(4 \quad 5\right)\begin{pmatrix} 1 \\ 2 \end{pmatrix} - 2 = 12$$

$$0 \leq \Delta c_4 \leq 12$$

$b)\ c_B\colon\ c_i \Rightarrow c_i + \Delta c_i$

$c_1 \Rightarrow c_1 + \Delta$

$$1)\ \hat{y} = (3+\Delta,\quad 9)\begin{pmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} = \left(1+\tfrac{\Delta}{3}+3,\quad -1-\tfrac{\Delta}{3}+6\right)$$

$$= \left(4+\tfrac{\Delta}{3},\ \ 5-\tfrac{\Delta}{3}\right)$$

$$\Delta_3 = \left(\hat{y},\ A_3\right) - 1 = \left(4+\tfrac{\Delta}{3}\right)2 + 3\left(5-\tfrac{\Delta}{3}\right) - 1$$

$$= 8+15-1-\tfrac{\Delta}{3} \geq 0$$

$$\frac{\Delta}{3} \leq 22, \qquad \Delta \leq 66$$

$$\Delta_4 = \left(\hat{y},\ A_4\right) = \left(4+\tfrac{\Delta}{3}\right)1 + 2\left(5-\tfrac{\Delta}{3}\right) - 2$$

$$= 12 - \frac{\Delta}{3} \geq 0$$

$$\Delta \leq 36 \qquad\qquad \therefore \Delta \leq 36$$

$$4+\frac{\Delta}{3}\geq 0 \qquad \Delta \geq -12$$

$$5-\frac{\Delta}{3}\geq 0 \qquad \Delta \leq 15$$

$$-12 \leq \Delta \leq 15$$

2) $\hat{y} = (3, \quad 9+\Delta)\begin{pmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

$$= \left(1+3+\tfrac{\Delta}{3},\ -1+6+\tfrac{2\Delta}{3}\right)$$

$$= \left(4+\tfrac{\Delta}{3},\ 5+\tfrac{2\Delta}{3}\right)$$

$$\left(\hat{y},\ A_3\right) = \left(4+\tfrac{\Delta}{3}\right)2 + \left(5+\tfrac{2\Delta}{3}\right)3 - 1 \geq 0$$

$$8+\frac{2\Delta}{3}+15+2\Delta \geq 1$$

$$\frac{8\Delta}{3} \geq -22$$

$$\Delta \geq \frac{-66}{8} = \frac{-33}{4}$$

$$\left(\hat{y}, A_4\right) = \left(4+\tfrac{\Delta}{3}\right)1 + \left(5+\tfrac{2\Delta}{3}\right)2 - 2$$

$$= 4 + \tfrac{\Delta}{3} + 10 + \tfrac{4\Delta}{3} - 2$$

$$= \tfrac{5\Delta}{3} + 12 \geq 0$$

$$5\Delta \geq -36$$

$$\Delta \geq \frac{-36}{5}$$

$$4+\frac{\Delta}{3} \geq 0, \qquad 5+\frac{2\Delta}{3} \geq 0$$

$$\Delta \geq -12, \qquad \Delta \geq \frac{-15}{2}$$

$$\infty > \Delta \geq \min\left\{\frac{-15}{2}, -12, \frac{-33}{4}, \frac{-36}{5}\right\}$$

$$= -7.2$$

$$y_1^* = \tfrac{1}{5}, y_2^* = \tfrac{2}{5}$$

$$x_3^* = 0$$

$$8x_1 + 3x_2 = 4$$

$$6x_1 + x_2 = 2$$

$$10x_1 = 2 \quad x_1 = \tfrac{1}{5}$$

$$x_2 = 2 - \tfrac{6}{5} = \tfrac{4}{5}$$